

# **NAVAL POSTGRADUATE SCHOOL**

## **Monterey, California**



## **THESIS**

**ESTIMATING HULL COATING THICKNESS  
DISTRIBUTIONS USING THE EM ALGORITHM**

by

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December 2000

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EM ALGORITHM**

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## ABSTRACT

The underwater hull coating system on surface ships is comprised anti-corrosive (AC) and anti-fouling (AF) paint. The AF layers are designed to wear away, continuously leaching cuprous oxide to inhibit marine growth. The thickness of the AF paint layers determines the expected service life of a coating system. Thus, it is important to assess the thickness of the AF layers to determine if the current hull coating system is sufficient. The Naval Ship Technical Manual (NSTM) provides specific guidelines as to how much AF paint should be applied. Unfortunately, the AF layers cannot be measured directly. The distribution of total paint thickness measurements is currently used as a proxy for the distribution of the thickness of the AF paint layers when determining if the existing coating system meets the hull coating requirements. We propose a remedy for this situation. A non-parametric maximum likelihood estimator for the cumulative distribution function of the AF layers, based on the EM algorithm, has been developed. Monte Carlo simulation is used to study the properties of this statistical approach for estimating the AF thickness. This model can be used to help decide if sufficient AF paint is on the underwater hull of a ship.



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## **LIST OF TERMS AND ABBREVIATIONS**

AC	Anti-corrosive sub-system
AF	Anti-fouling sub-system
cdf	Cumulative distribution function
CV	Conventionally powered aircraft carrier
CVN	Nuclear powered aircraft carrier
DFT	Dry film thickness
MSE	Mean squared error
NSTM	Naval Ship Technical Manual
PERA	Planning and Engineering for Repairs and Alteration command
pmf	Probability mass function
TT	Total thickness measurements



## **EXECUTIVE SUMMARY**

The underwater coating system of a ship is comprised anti-corrosive (AC) paint and anti-fouling (AF) paint. The AF paint is designed to slowly leach cuprous oxide, a toxin that prevents marine growth from attaching and living on the exterior of a ship's hull. To maintain a high concentration of cuprous oxide on the surface, today's anti-fouling paints wear away or ablate as the ship moves through the water, continuously exposing new paint with a high concentration of cuprous oxide. If sufficient AF paint remains after an operational cycle, then the coating system is salvageable and will only require simple repair and repainting. If, however, too much AF is worn away during the operational cycle, then not only is the ship at risk for hull fouling, but also at drydocking the entire coating system needs to be removed at great expense and the ship repainted. Thus, it is important to be able to assess the thickness of the AF layers.

The Naval Ship Technical Manual (NSTM) provides specific guidelines as to how much AF paint should be applied. Even in cases when AC measurements and total paint measurements are available, it is not possible to estimate the thickness of the AF layers directly. As a practical matter, it is impossible to measure the exact same point on a ship after the AC layers have been applied, and again after all of the total paint has been applied. The distribution of total paint thickness measurements is currently used as a proxy for the distribution of the thickness of the AF paint layers when determining if the existing coating system meets the hull coating requirements.

A non-parametric maximum likelihood estimator for the cumulative distribution function (cdf) of the AF layers, based on the EM algorithm, has been developed. This

thesis demonstrates how to use the new non-parametric estimator of the AF distribution thickness by providing a case study based on data from CVN-69. Monte Carlo simulation is used in this thesis to study the properties of this statistical approach for estimating the AF thickness. The simulation also allows for the study of the bias and the convergence properties of the EM algorithm for the estimated AF distribution.

There are several results from this thesis. The first is that an established process to estimate the anti-fouling distribution on US Navy ships has been evaluated and proven effective. This new method should replace the current proxy of total paint thickness for determining whether coating systems are salvageable during a ship's drydocking. The process will be available for the Department of the Navy to use when determining if simply repairing and applying additional coats of paint to an existing coating system is sufficient, or whether expensive paint removal and repainting costs are needed based on the estimated AF distribution. The second result of this thesis is that an analytical model now exists that may be utilized to test the effectiveness of whatever physical tool is developed, in the future, to directly measure the anti-fouling distribution. Finally, because this thesis studies a new approach for estimating deconvolutions, the results contribute to the general area of statistics.

## I. INTRODUCTION

The Navy currently spends about \$300 million per year associated with drydocking ships, of which approximately \$80 million is for ship hull preservation (Naval Surface Warfare Center, 1995). In an attempt reduce drydocking cost, the Navy is investigating ways to extend the lifetime of the underwater hull coating systems of its ships. To ensure cost savings, it is essential to apply sufficient paint at every drydocking so that coating systems are salvageable at the ship's next drydocking. The existing US Navy hull maintenance policy, as stated in the Naval Ship Technical Manual (NSTM), directs that all naval ships receive essentially the same underwater hull coating system without consideration to the ship's expected duration of operation or its anticipated hull maintenance requirements. The underwater coating system is comprised of 2-3 coats of anti-corrosive (AC) paint followed by 3-4 coats of anti-fouling (AF) paint. The AF paint is designed to slowly leach cuprous oxide, a toxin that prevents marine growth from attaching and living on the exterior of a ship's hull. Today's anti-fouling paints wear away or ablate as the ship moves through the water, continuously exposing new paint with a high concentration of cuprous oxide in order to maintain a high concentration of cuprous oxide on the surface. If sufficient AF paint remains after an operational cycle, then the coating system is salvageable and will only require simple repair and repainting. If, however, too much AF is worn away during the operational cycle, then not only is the ship at risk for hull fouling, but also at drydocking the entire coating system needs to be removed at great expense and the ship repainted. Thus, it is important to be able to assess the thickness of the AF layers. This thesis uses Monte Carlo simulation to study the properties of a statistical approach for assessing this thickness.

The US Navy's requirements for evaluating a hull coating system, as prescribed by NSTM Chapter 631, provides significant detail with respect to the physical evaluation of the paint's material condition. However, with respect to paint thickness, the NSTM only provides a range of appropriate paint thicknesses for each coat of paint. In accordance with NSTM guidelines, each coating system for a ship's hull should be identical, with all dry film thickness (DFT) measurements falling between a total coating thickness of 24 to 25 mils. This implicitly assumes that paint is applied uniformly over the entire hull, yet it well known that there is significant variability in hull coating thickness (Hinson, 1999). Each coat of paint is applied using spray guns, manually, while the ship is in drydock. Due to a variety of factors including the experience level of the painters, environmental conditions, and location of scaffolding, a coat of paint cannot be applied uniformly in a shipyard environment.

Since 1985, the Planning and Engineering for Repairs and Alterations command for US Navy aircraft carriers, PERA(CV), has closely monitored hull coating systems for all aircraft carriers. They inspect hull coating systems for all aircraft carriers at each drydock and routinely measure total thickness of the entire coating system at many locations over the hull after all the paint has been applied. The distribution of these total paint thickness measurements is used as a proxy for the distribution of the thickness of the AF paint layers when determining if the existing coating system meets the hull coating requirements. Research based on data collected by PERA(CV) also focuses on the total coating thickness distribution. Wimmer (1997) develops a model to predict the change of the total coating thickness as a function of the carrier's number of years at sea, number of hydro-washes, and number of underwater hull cleanings. Ellis (1999)

develops a method for estimating the mean thickness of an aircraft carrier hull's total coating system following two operational cycles. Hinson (1999) provides an analysis of paint wear as a function of hull paint smoothing. A method for estimating the thickness of the AF layers has not been addressed in the previously cited research.

It is actually the distribution of the AF paint thickness remaining on the hull that determines whether the current coating system may be salvaged. The anti-fouling paint layers must ablate in order to be effective, hence the measurement of the anti-fouling layers are the determining factor in the expected service life of a coating system. Even in cases when AC measurements and total paint measurements are available, it is not possible to estimate the thickness of the AF layers directly. As a practical matter, it is impossible to measure the exact same point on a ship after the AC layers have been applied, and again after all of the total paint has been applied. A solution to the problem of determining the thickness of the anti-fouling layers is to develop a physical measuring tool which may be placed on the outer surface of a ship and utilized to determine the thickness of the anti-fouling paint at that point directly. Until such a device is developed, a method is needed to estimate indirectly the distribution of the AF layers thickness. This thesis will focus on a new procedure for estimating the distribution of the AF layer thickness and will study its properties. Using data collected from the fleet's aircraft carriers, this analysis aims to provide a means to more accurately assess the hull coating system of a ship. A non-parametric maximum likelihood estimator for the cumulative distribution function (cdf) of the AF layers, based on the EM algorithm, has been developed (Whitaker, 2000) and will be analyzed in this thesis.

This study includes both a qualitative and quantitative analysis of the distribution of anti-fouling paint thickness of a hull coating system. Chapter II will provide a background of previous work, which will be used to validate the assumptions made when estimating the anti-fouling thickness distribution. In Chapter III, the EM algorithm is described as it pertains to estimating the anti-fouling paint distribution. As an illustration, a case study utilizing actual data will be analyzed. In Chapter IV, the EM algorithm is applied to randomly generated data to study the properties of the anti-fouling estimator. Finally, Chapter V concludes the analysis with recommendations and discussion.

## **II. COATING SYSTEM PROPERTIES**

Currently, the primary method of evaluating the condition of a hull coating system is to collect dry film thickness (DFT) measurements of the total paint thickness from random locations on the hull. These total paint thickness data sets have been used as a proxy for the distribution of the AF paint, though it is actually the distribution of the remaining AF paint which is the determinant of whether sufficient paint remains on the hull to salvage the current coating system. Intuitively, the easiest way to determine anti-fouling thickness at a specific location is to measure the anti-corrosive (AC) paint thickness prior to applying the anti-fouling paint and then re-measure the total paint thickness at the exact same location after the anti-fouling paint is applied. However, in practice it is virtually impossible to replicate the total thickness measurements at the exact location that the anti-corrosive measurements were taken. Wimmer (1997) and Ellis (1999) show that there is a large variance in hull coating thickness in just a small area; with such a variance in total thickness measurements, even a slight error in replication of the anti-corrosive and total paint measurement locations may produce very different results. Hence, the distribution of the anti-fouling sub-system must be found from the distribution of the anti-corrosive inner sub-system and the distribution of the total coating thickness.

Wimmer (1997) and Ellis (1999) provide models that predict the thinning of ablative anti-fouling layers, as a function of the operational cycle such as years at sea and number of hull cleanings, using DFT measurements. These predicted distributions provide information not previously known about the wear of coating systems during one to two operational cycles.

## A. HULL COATING THICKNESS PROPERTIES

### 1. Initial Variability

An initial hull coating system consists of multiple coats of anti-corrosive and anti-fouling paints applied to a hull that has been sandblasted to bare metal. Table 1 provides the required thickness for each coat, as prescribed by NSTM, Chapter 631.

Coating Type		Thickness
1st coat	Epoxy primer	2-3 mils
2nd coat	Anti-corrosive paint	5 mils
3rd coat	Anti-corrosive paint	5 mils
4th coat	Anti-fouling paint	4 mils
5th coat	Anti-fouling paint	4 mils
6th coat	Anti-fouling paint	4 mils

**Table 1. NSTM Prescribed Hull Coating System Thickness (Total=24-25 mils).**

Each coat of paint is applied using spray guns, manually, while the ship is in drydock.

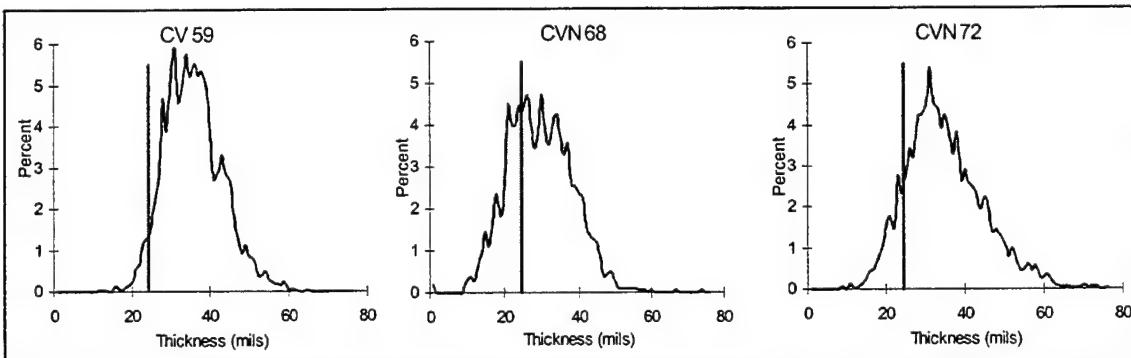
Due to various factors including the experience level of the painters, environmental conditions, and location of scaffolding, a coat of paint cannot be applied uniformly at its prescribed thickness in a shipyard environment. In reality, the thickness of a single coat of paint varies considerably within a very small area. Additionally, the variability of a coating system's thickness increases as each additional coat of paint is applied. (Hinson, 1999)

PERA(CV) data is used to demonstrate the large variability in initial coating thickness. This data consists of dry film thickness (DFT) measurements collected from various randomly selected locations on the hulls of three ships. Table 2 provides summary statistics of total paint thickness for the hull coating systems of the USS Forrestal (CV 59), the USS Nimitz (CVN 68), and the USS Lincoln (CVN 72) immediately following application.

Ship	Minimum	1st Quartile	Median	Mean	3rd Quartile	Maximum	Sample Size
USS Forrestal (CV 59)	10.6	29.1	33.8	34.3	38.8	63.2	3030
USS Nimitz (CVN 68)	8.3	21.9	27.9	28.2	34.1	72.9	2099
USS Roosevelt (CVN 72)	7.7	26.6	32.1	33.4	39.2	82	4095

**Table 2. Summary Statistics of Three Freshly Painted Hull Coating Systems.**

In accordance with NSTM standards, each of these coating systems should be identical, with all DFT measurements falling between a total thickness of 24 to 25 mils. However, the smallest range of paint thickness, as shown in Table 2, is 52.6 mils for CV 59. The largest range of paint thickness is 74.3 mils for CVN 72.



**Figure 1. Hull Coating System Thickness After Initial Paint Application. The Vertical Lines Identify the NSTM Prescribed Thickness Standard, 24-25 mils.**

As illustrated in Figure 1, fewer than ten percent of each ship's DFT measurements actually fall within the prescribed range.

## 2. Initial Hull Paint Thickness Distributions

Wimmer (1997) and Ellis (1999) show that the anti-corrosive sub-system and the total coating thickness distributions to be non-parametric and also asymmetric. The Kolmogorov-Smirnov Goodness-of-Fit test rejects the null hypothesis of normality for CV 59, CVN 68, and CVN 72 with p-values less than 0.000001. Other families of distributions are fit to the three data sets to determine if a single family of distributions may characterize all three freshly applied coating systems. The closest fit appears to be

the Gamma distribution. Unfortunately, the Kolmogorov-Smirnov Goodness-of-Fit test rejects the null hypothesis that the distribution is Gamma with p-values less than 0.0001.

Taking into account the large sample sizes available, a feasible alternative is to use a non-parametric estimator for the distribution of hull paint thickness. The most straightforward non-parametric estimator is the empirical cumulative distribution function (cdf). For any value  $x$ , the empirical cdf gives the proportion of measurements that are less than or equal to  $x$ , and it may be computed directly. Further, this estimator has a robust property in that its performance is not sensitive to the shape of the true distribution. This is important since there has been shown that a ship's paint thickness distribution often may not be adequately modeled by a particular family of distributions. The empirical cdf's for CV 59, CVN 68, and CVN 72 are provided in Figure 2.

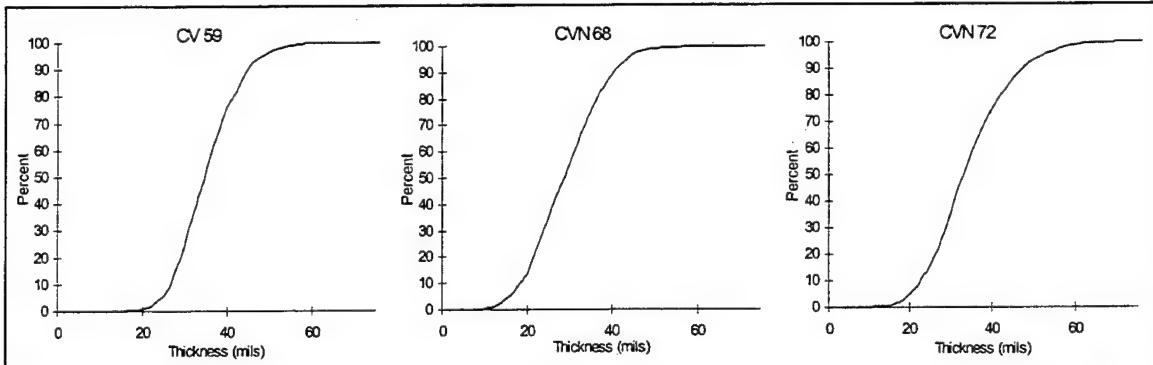


Figure 2. Empirical cdf's for Three Freshly Painted Hull Coating Systems.

## B. ESTIMATING THE AF DISTRIBUTION

PERA(CV) routinely collects total thickness measurements after all the paint has been applied. This quantity will be referred to using the notation of TT. These total thickness measurements may be taken at any time, thus they are considered inexpensive and may be plentiful, if desired. There exists limited sample data from measurements taken after just the AC layers have been applied, since collection of these measurements was not necessary previously. Anti-corrosive thickness measurements must be taken

after the AC layers are applied and before the AF layers are applied, thus AC measurements must be taken at the initial hull painting. Even in cases when AC measurements and TT paint measurements are available, it is not possible to estimate the AF layers directly because the measurements are taken at different and independent locations on the hull. In reality, it is impossible to replicate the total thickness measurements at the precise location that the anti-corrosive measurements were taken. With the large variance in coating thickness, even a slight error in location may produce very different DFT measurements. Thus, the distribution of the anti-fouling sub-system must be found from the deconvolution of the distributions of the anti-corrosive sub-system and the total coating system.

There has been considerable research in the area of estimating deconvolutions. For example, Stefanski and Bay (1996) study a simulation extrapolation deconvolution of finite population cumulative distribution functions. Medgyessy (1977) provides a detailed overview of deconvolutions. Previous work in the area of deconvolutions, however, often assumes symmetry or a parametric form for the underlying distributions. Wimmer (1997) and Ellis (1999), however, show it is not possible to adequately model hull coating distributions by a single parametric family and that the distributions are not symmetric. Hence, none of the previous approaches for deconvolutions are appropriate for estimating the distribution of AF thickness. As a result of these limitations, other systematic methods have been developed to estimate the distribution of AF.

Let the random variables T, AF, and AC represent total, anti-fouling, and anti-corrosive thickness at a particular location on the hull. Then,  $T = AF + AC$ . Wimmer (1997) provides an ad-hoc estimator for the cdf of the anti-fouling thickness,  $F_{AC}$ . We do

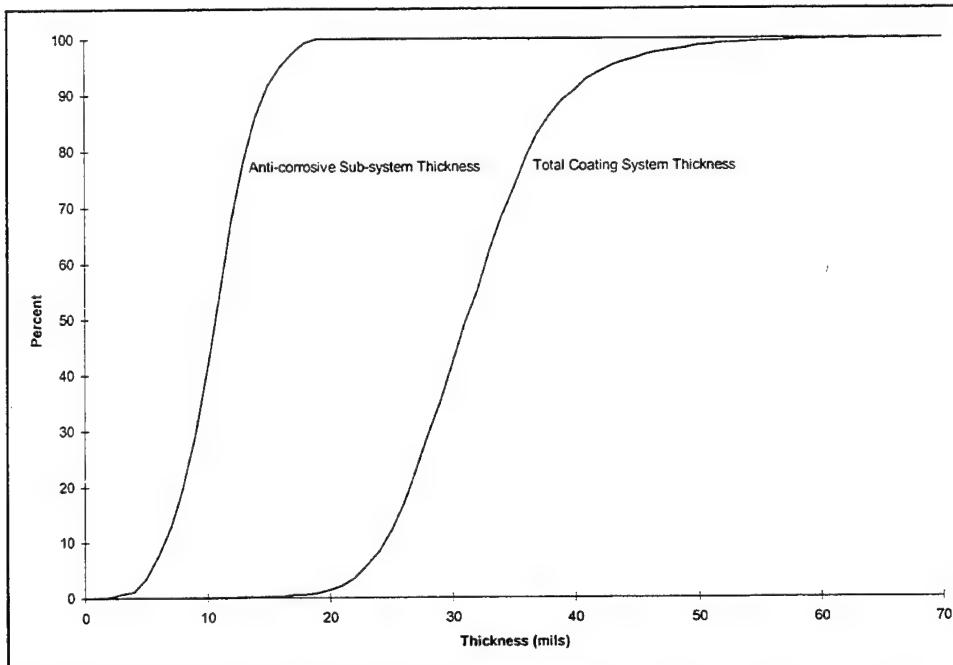
not know whether a parametric family may model the cdf of the anti-corrosive thickness,  $F_{AC}$ . Figure 1 suggests that the cdf of the total thickness,  $F_T$ , is not symmetric. Wimmer develops the following *ad hoc* estimator for  $F_{AF}$ . Let  $T_{max}$  be an upper bound for the total coating thickness and  $0 = a_1 < a_2 < \dots < a_N = T_{max}$  be  $N$  equally spaced values between 0 and  $T_{max}$ . Wimmer's method approximates  $F_{AF}$ ,  $F_{AC}$  and  $F_T$  by discrete versions of these distributions. With this simplification and the assumption that at a given location the anti-corrosive thickness AC and the anti-fouling thickness AF are independent:

$$\begin{aligned}
 F_T(a_1) &= F_{AC}(a_1) P(AF = a_1), \\
 F_T(a_2) &= F_{AC}(a_1) P(AF = a_2) + F_{AC}(a_2) P(AF = a_1), \\
 &\vdots \\
 F_T(a_i) &= \sum_{j=1}^i F_{AC}(a_j) P(AF = a_{i-j+1}), \\
 &\vdots \\
 &\vdots \\
 F_T(a_N) &= \sum_{j=1}^N F_{AC}(a_j) P(AF = a_{N-j+1}).
 \end{aligned} \tag{2.1}$$

Replacing  $F_T$  and  $F_{AC}$  with the empirical cdf's  $\hat{F}_T$  and  $\hat{F}_{AC}$  and solving the system of linear equations (2.1), we obtain estimates  $\hat{P}(AF = a_i)$  of  $P(AF = a_i)$  from which we can compute:

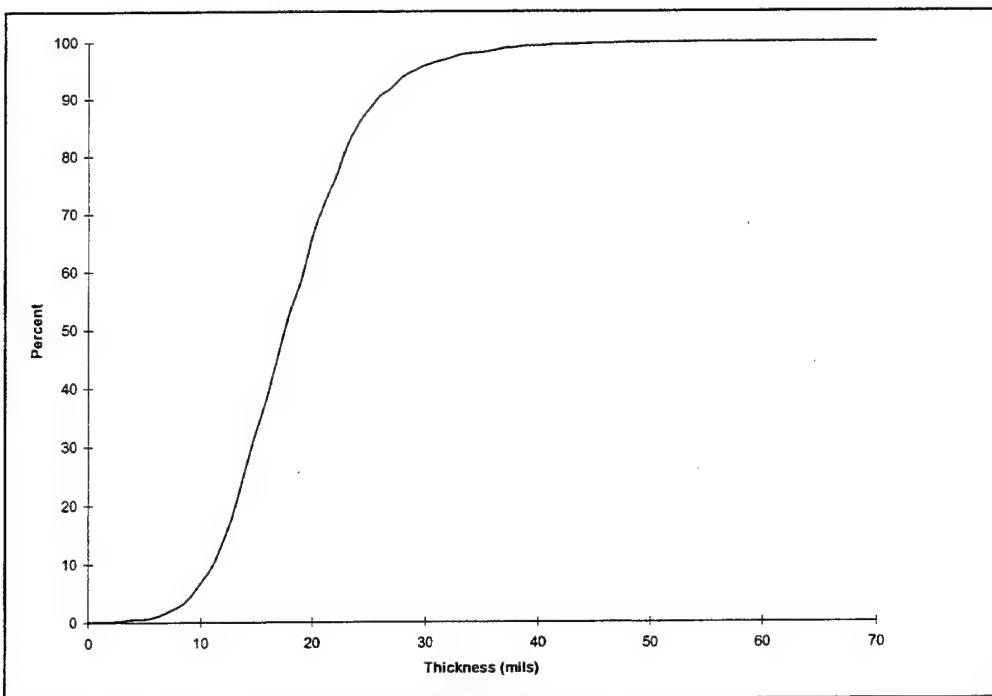
$$\hat{F}_{AF}(x) = \sum_{\{i : a_i \leq x\}} \hat{P}(AF = a_i).$$

As an example of the deconvolution process using the Wimmer method, assume that a hull coating system has the anti-corrosive empirical cdf and a total paint thickness empirical cdf illustrated in Figure 3.



**Figure 3. Empirical cdf's of an AC Sub-System and a Total Coating System.**

For simplicity, the distribution of AF is estimated by partitioning [0, 70 mils] into equally spaced values incremented by one mil. The Wimmer ad-hoc method is then applied to the data set displayed in Figure 3. The resultant estimated cdf of the anti-fouling thickness distribution, shown in Figure 4, provides a comprehensive view of the entire anti-fouling sub-system.



**Figure 4. Estimated Anti-Fouling Thickness cdf.**

Now, instead of using the total coating system thickness as a proxy for the thickness distribution of the anti-fouling sub-system, the estimate of the anti-fouling sub-system may be evaluated directly. For example, from Figure 4, approximately ten percent of the anti-fouling sub-system is below the NSTM prescribed thickness of 12 mils. Therefore, a more informed evaluation may be made concerning the status of the anti-fouling sub-system. This method of evaluating paint sufficiency provides a tremendous advantage over the current "fixed total thickness" method, since it estimates the actual anti-fouling paint thickness distribution. The current method relies upon the unrealistic assumption that every ship in the US Navy has an identical anti-corrosive paint sub-system.

A non-parametric maximum likelihood estimator for the cumulative distribution function (cdf) of the AF layers, based on the EM algorithm, has been developed (Whitaker, 2000). Like Wimmer's ad-hoc approach this estimator is non-parametric. It is also based on anti-corrosive and total thicknesses, independent of each other, that can

be modeled as identical and independent with respective distributions  $F_{AC}$  and  $F_T$ . There are two main features of the Whitaker method. First, note that Wimmer (1997) and Ellis (1999) show that the anti-corrosive sub-system and the total coating thickness distributions cannot be adequately modeled by a single parametric family of distributions. In the Whitaker estimator, the anti-fouling sub-system distribution is non-parametric. Non-parametric estimation is feasible for hull coating systems due to the large sample sizes that are easily obtained. The second feature involves the independence of the anti-corrosive and total data sets. This also appears appropriate since these data sets are comprised of simple random samples taken at a variety of locations on the ship's hull. Wimmer (1997) demonstrates that the location of the measurements on the hull, whether taken forward or aft on the starboard or port side, does not effect the distribution of the total thickness. Although the Whitaker estimator was developed specifically for estimating anti-fouling sub-system thickness, the approach is general and can be used in any application for non-parametric estimation.

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### III. ANTI-FOULING DISTRIBUTION ESTIMATOR

The EM algorithm provides a numerical method for computing maximum likelihood estimators (MLEs). This algorithm is widely used in incomplete data problems such as estimating survival functions in reliability based upon censored data. Were complete data available for these problems, simply computed expressions for the MLEs might be used. The EM algorithm is used when the likelihood, based on incomplete data, cannot be maximized easily. The EM algorithm applies two estimation schemes in repeated alternative fashion. First, estimating the expected values of the complete observations given the available data; then, using the previous iteration approximation of the MLE, the algorithm updates the MLE using the estimates of the expected values as if they were complete observations. This completes a single cycle of the algorithm.

One limitation of the EM algorithm is that upon termination, the approximate MLE is not always near the actual MLE. As reported by Rosenberg and Gail (1989), when several stationary points exist, convergence to a particular stationary point depends upon the choice of starting point. However, if the algorithm is coded such that the parameters are restricted to the interior of the parameter space, then the EM algorithm will converge to a local maximum of the likelihood function. The algorithm sometimes may take many iterations to converge to these local maxima. It is essential to select an appropriate initial starting solution. The EM algorithm is known to be extremely slow to converge. Speeding up the EM algorithm has been a topic of interest since its formulation, but thus far it has only been achieved by sacrificing the simplicity or stability of the algorithm (Meng, 1995).

## A. SYSTEMATIC INITIAL ESTIMATE AD-HOC METHOD

As described in Chapter II, Wimmer (1997) provides an ad-hoc estimator for the cdf of the anti-fouling thickness. The advantage of the Wimmer method is that it is robust to non-symmetric and non-parametric data. In theory, Wimmer's ad-hoc estimator appears useful in determining an appropriate initial starting solution for the application of the EM algorithm. In practice, however, it does not always yield the probability mass function,  $P(AF=a_i)$ . Because AC and T measurements are independent, some actual AC thickness measurements are larger than some of the total thickness measurements, yet the Wimmer method makes no provision for this. For example, summary information for CVN 69 anti-corrosive and total thickness data sets is presented in Table 3.

Paint Type	Minimum	1 <sup>st</sup> Quartile	Median	Mean	3rd Quartile	Maximum	Sample size
Anti-Corrosive	7	12	13	13.5	16	22	127
Total	19	36	40	40.2	44	70	1127

**Table 3. Summary Statistics of Initial Paint Thickness (mils) for CVN 69.**

Table 3 shows the maximum measurement for the anti-corrosive thickness is 22 mils and the corresponding minimum measurement for total thickness is 19 mils. This simple example illustrates a weakness in the Wimmer ad-hoc method since there is no way of jointly accounting for the minimum total measurements and the maximum anti-corrosive measurements. Thus, in practice, the Wimmer method is often unable to provide feasible initial starting values for the EM algorithm.

## B. LEAST SQUARES INITIAL ESTIMATE

Another means of estimating the cumulative distribution function of the anti-fouling thickness distribution is to take a least square approach. Anti-corrosive and total thickness measurements are rounded to the nearest integer to simplify matrix manipulation in the "Least Squares Initial Estimate" shown below. In addition, an upper limit of 70 mils is used for T since this value is significantly above the prescribed

NSTM standard for total thickness of 24 to 25 mils. Because  $T = AC + AF$  and with the independence of AC and AF, we have:

$$\begin{aligned} P(T \leq 0) &= P(AC \leq 0)P(AF = 0), \\ P(T \leq 1) &= P(AC \leq 1)P(AF = 0) + P(AC \leq 0)P(AF = 1), \\ P(T \leq 2) &= P(AC \leq 2)P(AF = 0) + P(AC \leq 1)P(AF = 1) + P(AC \leq 0)P(AF = 2), \end{aligned}$$

$$P(T \leq 140) = P(AC \leq 140)P(AF = 0) + P(AC \leq 139)P(AF = 1) + \dots + P(AC \leq 0)P(AF = 140).$$

These equations may be written in matrix notation as:

$$\begin{bmatrix} P(T \leq 0) \\ P(T \leq 1) \\ P(T \leq 2) \\ \vdots \\ P(T \leq 140) \end{bmatrix} = \begin{bmatrix} P(AC \leq 0) & 0 & 0 & 0 & \dots & 0 \\ P(AC \leq 1) & P(AC \leq 0) & 0 & 0 & \dots & 0 \\ P(AC \leq 2) & P(AC \leq 1) & P(AC \leq 0) & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ P(AC \leq 140) & P(AC \leq 139) & \dots & P(AC \leq 0) & & \end{bmatrix} * \begin{bmatrix} P(AF = 0) \\ P(AF = 1) \\ P(AF = 2) \\ \vdots \\ P(AF = 70) \end{bmatrix},$$

or equivalently:

$$Y = XB \quad (3.1)$$

In equation 3.1, Y is a column vector containing the cdf of T, X is the 141 x 71 triangular matrix whose entries are the cdf of AC, and B is the column vector containing the probability mass function (pmf) of AF. The Y vector is estimated using the empirical cdf of the total thickness data. The X matrix is estimated by the empirical cdf of the AC data. Then, the least square solution which satisfies:  $\min_B \|Y-XB\|$  is found for B. The least squares solution for the initial estimate of the anti-fouling distribution, the B matrix, is then found by solving:  $B = (X^T * X)^{-1} * Y$ . Like the Wimmer method, the initial least squares solution also does not always yield the probability mass function,  $P(AF=a_i)$ . Due to the independence of the anti-corrosive thickness and total thickness data, it is possible for the least square solution of  $P(AF=a_i)$ , as described, to be less than zero. There are also cases where  $P(AF=a_i) > 0$  is possible, and yet this is not feasible based on known TT

data. It is necessary to remove negative probabilities and infeasible initial estimates when they appear in the least square solution, followed by a re-scaling of the probability mass function in order to ensure a sensible initial guess for  $P(AF=a_i)$ . This estimator is ad-hoc, but when it is used in conjunction with the Whitaker model the speed of convergence of the EM algorithm is greatly improved.

### C. WHITAKER (2000) MODEL

At any particular location on the ship's hull,  $AC + AF = T$ , for  $AC$  and  $AF$  independent random variables. Hence, the distribution of  $T$ ,  $F_T$ , is the convolution of  $F_{AC}$  and  $F_{AF}$ . Without loss of generality, let the support of  $F_{AC}$  and  $F_T$  be the integers  $\{0, 1, 2, \dots, AC_{\max}\}$  and  $\{0, 1, 2, \dots, T_{\max}\}$  respectively. The  $AC$  and  $T$  measurements are taken at different locations across the hull of a ship. Let  $N_{AC}$  be the size of a simple random sample of  $AC$  measurements. It is independent of the simple random sample of size  $N_T$  consisting of  $T$  measurements. Additionally, let  $N_i$  be the number of  $AC$  observations out of  $N_{AC}$  with value  $i = 1, 2, \dots, AC_{\max}$  and let  $M_i$  be the number of  $T$  observations out of  $N_T$  with the value  $i = 1, 2, \dots, T_{\max}$ . Then, the joint likelihood of  $N_{AC}$  independent, identically distributed observations from  $F_{AC}$  and  $N_T$  independent, identically distributed observations from  $F_T$  may be expressed as:

$$L = \prod_{i=0}^{AC_{\max}} P(AC = i)^{N_i} \prod_{i=0}^{T_{\max}} P(T = i)^{M_i}$$

To find estimators for the distributions of  $F_{AC}$ ,  $F_{AF}$ , and  $F_T$ ,  $L$  is maximized subject to the constraints that  $F_T$  is the convolution of  $F_{AC}$  and  $F_{AF}$ . In general, there is no closed form solution to this problem. However, there is a simple solution to the deconvolution problem using the EM algorithm. Here, both the  $T$  measurements and the  $AC$  measurements are viewed as incomplete observations from two independent

“complete” data sets that include AF measurements corresponding to each T and AC measurement.

The first “complete” data set used in the EM algorithm, named Data Set #1, augments the observed T measurements with AF measurements. This complete data can be summarized as  $X_{ij}$ , with  $i = 1, 2, \dots, AF_{\max}$  and  $j = 1, 2, \dots, T_{\max}$ , where  $X_{ij}$  is the number of observations in Data Set #1 with AF=i and T=j. Hence,

$$\sum_{i=0}^{AF_{\max}} \sum_{j=0}^{T_{\max}} X_{ij} = N_T,$$

and  $\sum_{i=0}^{AF_{\max}} X_{ij}$  is the number of observed T with a value of j.

The second “complete” data set, named Data Set #2, augments the AC measurements with corresponding AF measurements. Let  $Y_{ij}$  be the number of observations in Data Set #2 summarized as  $Y_{ij}$ , with  $i = 1, 2, \dots, AF_{\max}$  and  $j = 1, 2, \dots, T_{\max}$ , where  $Y_{ij}$  is the number of observations in Data Set #2 with AF=i and T=j.

Hence,  $\sum_{i=0}^{AF_{\max}} \sum_{j=0}^{AC_{\max}} Y_{ij} = N_{AC}$  and  $\sum_{j=i=k}^{AF_{\max}} Y_{ij}$  is the number of observed AC values which are equal to k.

Let  $\theta_0, \dots, \theta_{AF_{\max}}$  be the pmf of AF and  $\tau_0, \dots, \tau_{AC_{\max}}$  be the pmf of AC. Then, the EM algorithm may be applied to solve the deconvolution problem. The initial iteration values to be used as complete data are defined as Data Set #1,  $X_{ij}^{(1)}$ , and Data Set #2,  $Y_{ij}^{(1)}$ . First, the E step of the EM algorithm involves matrix manipulation to compute  $X_{ij}^{(1)}$  and  $Y_{ij}^{(1)}$ . The expected values of the “complete” data, given the observed data, is:

$$E[X_{ij} \mid \text{observed T}] = X_{\cdot j} \frac{P(AF = i, AC = j-i)}{P(T = j)}$$

$$= X_{\cdot j} \frac{\theta_i \tau_{j-i}}{\sum_{k=0}^j \theta_k \tau_{j-k}},$$

where  $X_{\cdot j} = \sum_{i=1}^{AF_{\max}} X_{ij}$ .

Similarly, for Data Set #2:

$$E[Y_{ij} \mid \text{observed AC}] = \left( \sum_{k-l=i-j}^{AC_{\max}} Y_{k,l} \right) \frac{\theta_i}{\sum_{i=0}^{n-j} \theta_i}.$$

Substituting our initial guess  $\theta_0^{(0)}, \dots, \theta_{AF_{\max}}^{(0)}$  and  $\tau_0^{(0)}, \dots, \tau_{AC_{\max}}^{(0)}$  for the appropriate marginal distributions produces the first iteration estimates of the complete data

$X_{ij}^{(1)}$  and  $Y_{ij}^{(1)}$ .

The M part of the EM Algorithm is designed to compute  $\theta_i^{(1)}$  and  $\tau_k^{(1)}$ , the first iteration estimates of the AF and AC distributions. Let  $\{p_{ij}\}$  be the joint pmf of AF and AC. Based on complete data,  $X_{ij}$  and  $Y_{ij}$ , the maximum likelihood estimators  $\hat{p}_{ij}, \hat{\theta}_i, \hat{\tau}_k$  of  $p_{ij}, \theta_i, \tau_k$  respectively are computed as follows:

$$\hat{p}_{ij} = \frac{X_{ij} + Y_{ij}}{N_{AF} + N_{AC}},$$

$$\hat{\theta}_i = \sum_{j=i}^m \hat{p}_{ij}, \text{ and}$$

$$\hat{\tau}_k = \sum_{j-i=k} \hat{p}_{ij}.$$

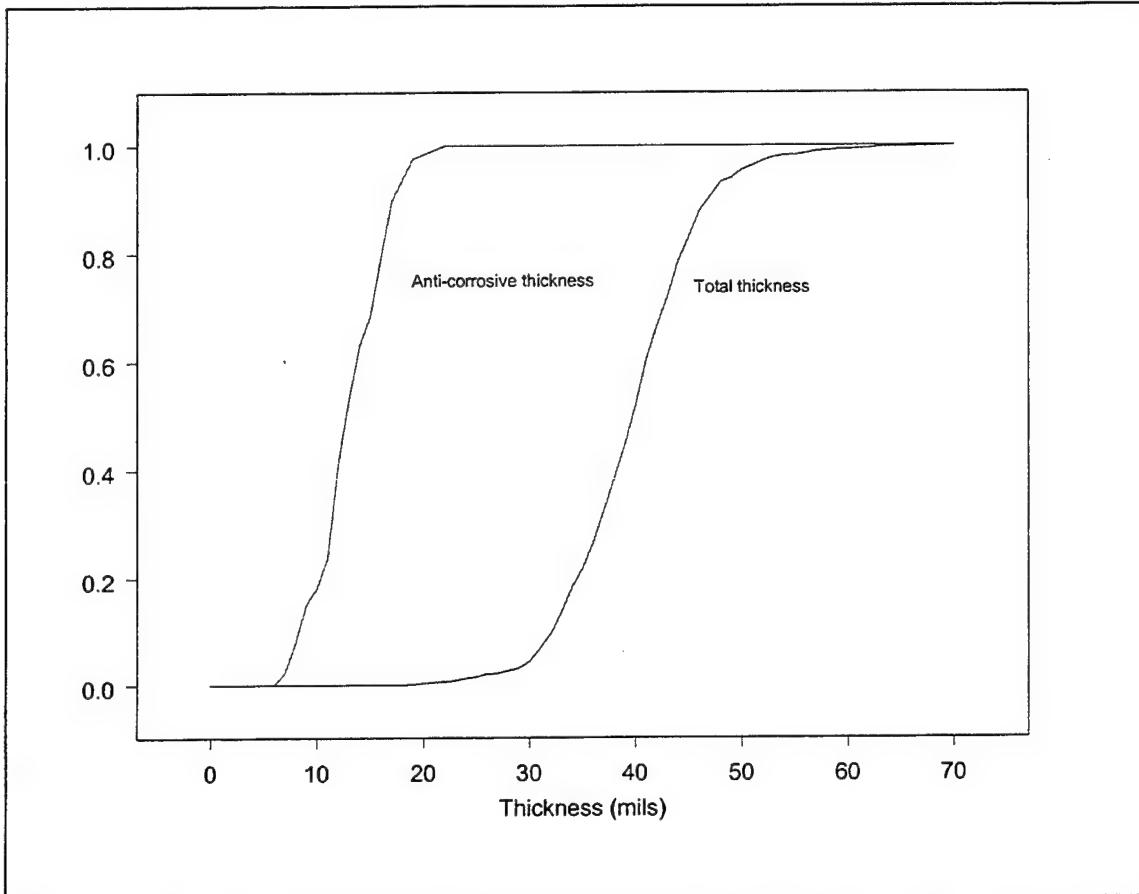
Thus, the first iteration estimates are found by substituting the estimated complete data,

$X_{ij}^{(1)}$  and  $Y_{ij}^{(1)}$ , for  $X_{ij}$  and  $Y_{ij}$ .

The EM algorithm requires an initial guess at the marginal distribution of AF. Due to the known lack of speed of the EM algorithm, the initial guess to be utilized during simulations for this thesis is based on a least squares estimate discussed in Chapter II. At the  $k^{\text{th}}$  step of the EM algorithm, the E step will compute  $X_{ij}^{(k)}$  and  $Y_{ij}^{(k)}$  and then the M step computes  $\hat{p}_{ij}^{(k)}, \hat{\theta}_i^{(k)}, \hat{\tau}_k^{(k)}$ . The algorithm then iterates back through E step until sufficient terminating conditions are reached giving the final estimates of output of  $\hat{\theta}_i, \hat{\tau}_i$ .

#### D. AN ILLUSTRATION

In 1985 technicians collected baseline anti-corrosive and total paint thickness measurements on the USS Dwight D. Eisenhower (CVN 69). Summary statistics for this data set are provided in Table 3. This data set provides a realistic application and illustration of the Whitaker model for an estimate of the anti-fouling thickness distribution. Figure 5 shows the empirical cdf of the AC and TT distributions.

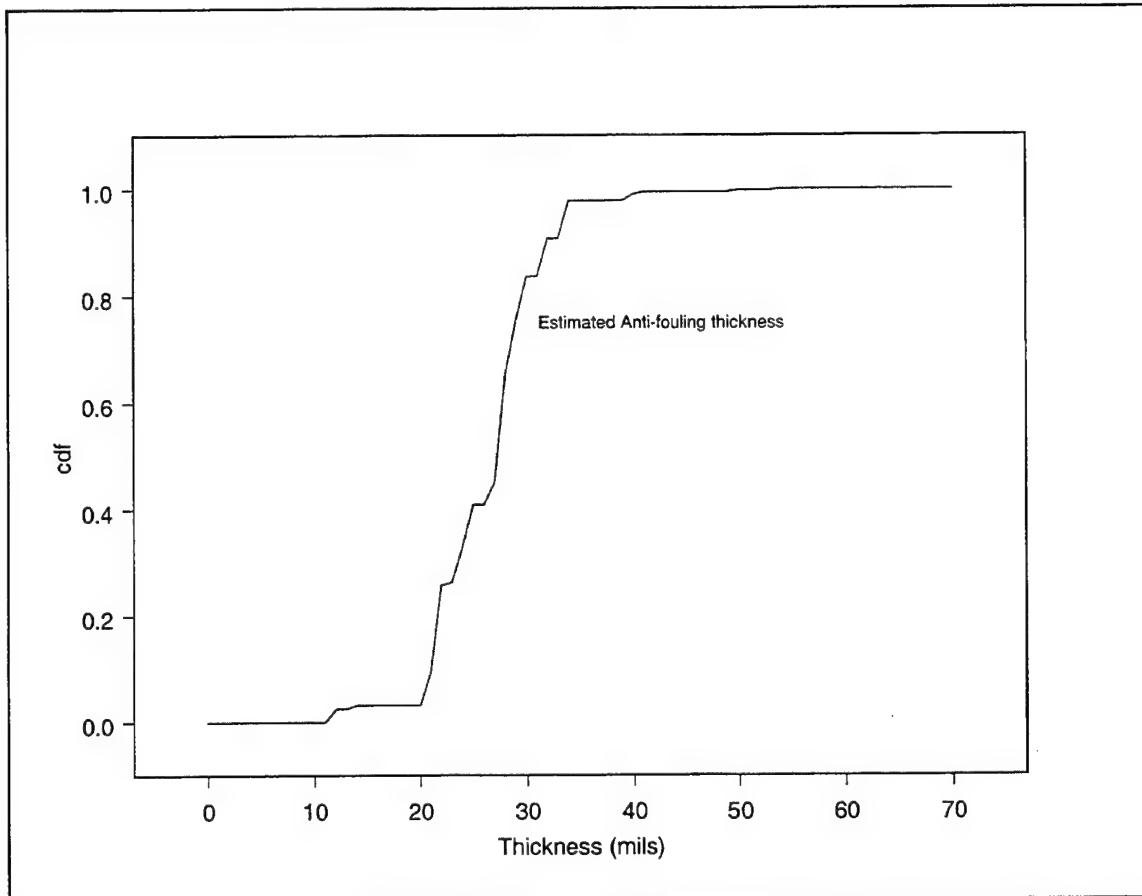


**Figure 5. The Empirical cdf of the Anti-Corrosive and Total Paint Thickness for USS Dwight D. Eisenhower (CVN 69) in 1985.**

Current NSTM guidelines require a total coating measurement of 24-25 mils and an anti-corrosive sub-system measurement of 12-13 mils. Initial analysis of the total data set shows that 98.5% of the total thickness measurements are above this prescribed range. Additionally, 52.7% of the anti-corrosive data set measurements are above the required range.

The Whitaker method is applied to develop an estimate for the distribution of the anti-fouling paint thickness. The central question is whether there is sufficient anti-fouling paint applied to the carrier hull. If the answer to this question is no, then the

existing paint must be removed and re-applied, at great expense. Figure 6 shows the estimate for the distribution of the anti-fouling paint thickness.



**Figure 6. The Estimated cdf of the Anti-Fouling Paint Thickness for USS Dwight D. Eisenhower (CVN 69), Based on Known Anti-Corrosive and Total Thickness Data.**

The NSTM prescribes an anti-fouling thickness of 12 mils. The estimate for the distribution of the anti-fouling paint shows that over 97% of the AF distribution exceeds this requirement. Additionally, from the Whitaker estimator it is apparent that over 67% of the anti-fouling paint on the USS Eisenhower is more than 24 mils in thickness, which is twice the required amount. Thus, in this illustration, the current coating system is deemed sufficient.

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## **IV. EM ALGORITHM SIMULATIONS**

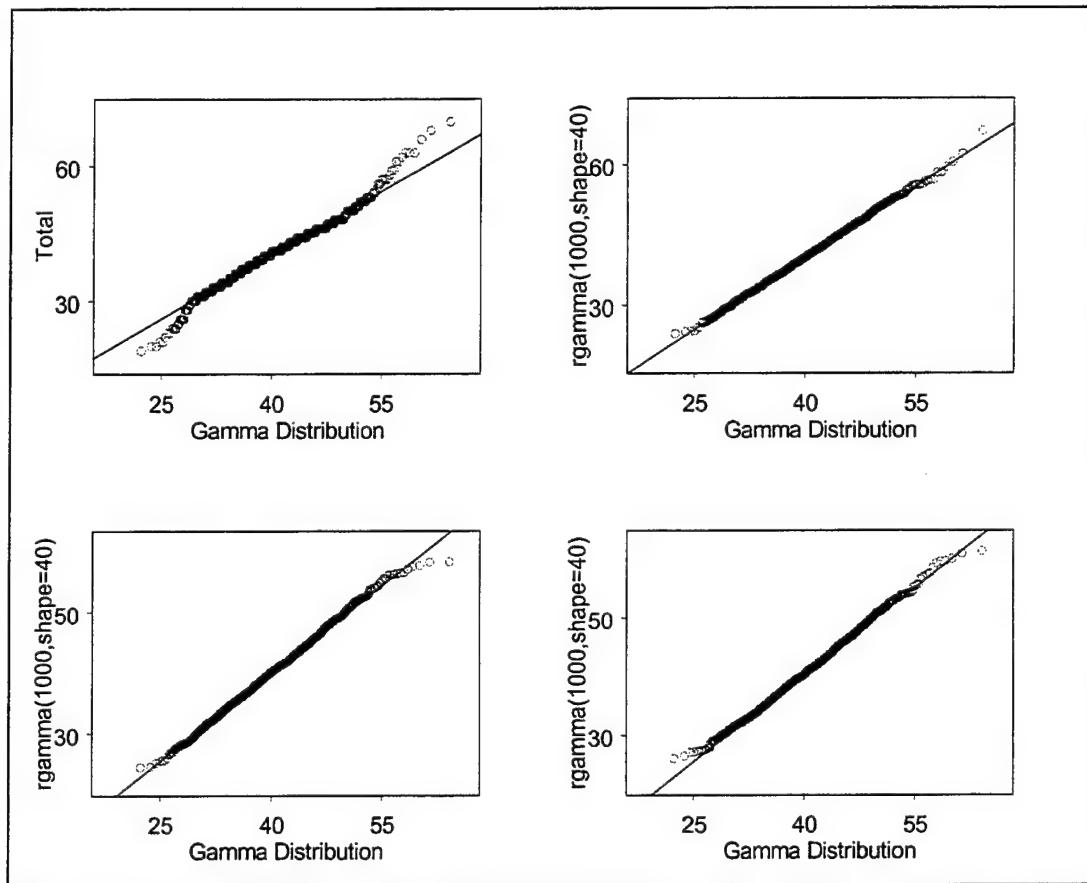
The most important question for hull coating systems is “Does the current hull coating possess sufficient anti-fouling paint to be maintained or enhanced at the ship’s next drydock, or must the entire coating system be replaced?” Any model to estimate the anti-fouling thickness distribution must consider some key elements. First, the estimator must be able to predict the AF thickness distribution for any hull coating system. As illustrated in Chapter II, the estimator must be robust to the shape of the AF and T distributions. Thus, it is necessary to study the performance of the estimator for different types of underlying AF, AC, and T distributions. Secondly, because AC measurements are more difficult and “expensive” to obtain than measurements of total thickness, it is also important to study how small numbers of AC measurements affect the estimator. Finally, because the AF distribution estimator does not have a closed form, but is computed using an algorithm, convergence properties of the algorithm must be studied. In particular, how many iterations are needed for convergence.

This chapter develops a set of simulations that quantifies the effectiveness of the Whitaker estimator for the distribution of the anti-fouling paint thickness. The simulations are constructed to explore realistic scenarios.

### **A. UNDERLYING T, AC, AND AF DISTRIBUTIONS**

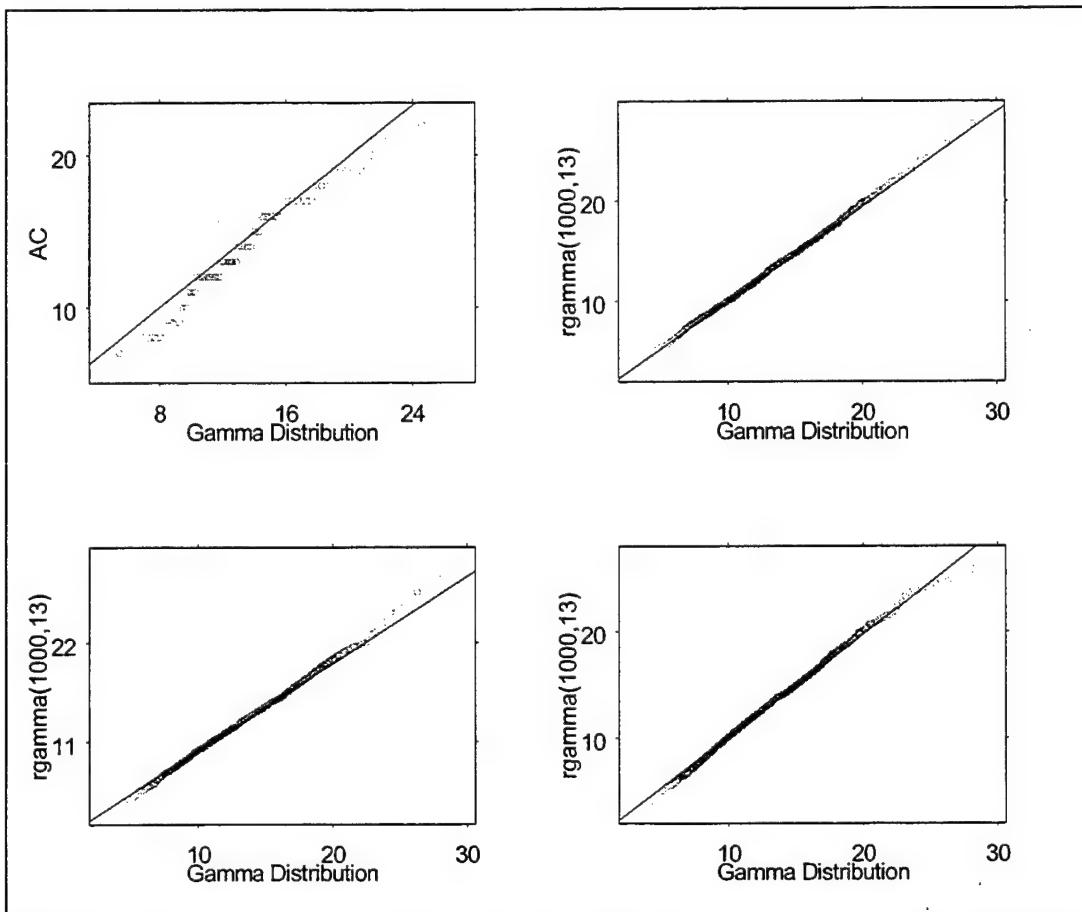
In order to simulate realistic data sets, actual TT and AC data sets are taken from CV 59, CVN 68, and CVN 72 and analyzed to choose underlying distributions. The gamma distribution and the normal distribution appear to be the closest fit to the available data sets. From these data sets, parameters to start randomly generating data from are estimated. For example, the initial gamma “rate” and “shape” parameters are selected to be as realistic as possible for both the TT and the AC data sets. Utilizing data from the

CVN 69 data sets, Figure 7 indicates an appropriate initial gamma shape parameter for total data to be 40, with a rate parameter of 1.



**Figure 7. Actual TT Data from CVN 69 in 1985 as Compared to 1000 Samples of Randomly Generated Gamma Data With a Shape Parameter of 40.**

Figure 8 suggests an appropriate initial gamma shape parameter for AC data to be 13 and a rate parameter of 1. These initial parameters are then varied to study the effect different distributions have on the estimator procedure.



**Figure 8. Actual AC Data from CVN 69 in 1985 as Compared to 1000 Samples of Randomly Generated Gamma Data With a Shape Parameter of 40.**

Similarly, initial parameters are chosen for data generation for other parametric families. For example, utilizing data from the 1985 CVN 69 data sets, an appropriate initial normal mean for total data is found to be 40.17, with a standard deviation of 6.31. Additionally, an appropriate initial normal mean for AC data is found to be 13.49, with a standard deviation of 3.28. S-Plus is the tool utilized to randomly generate data for the simulations.

In order to test the effectiveness of the estimator, the anti-fouling thickness distribution must be known. Hence, we generate data from a variety of distributions, which are closed under convolution, in order to evaluate the estimator. The normal and

the gamma distributions provide examples of such distributions. For the normal distribution, since  $AF + AC = T$ , if  $AF \sim \text{Normal}(\mu_{AF}, \sigma^2_{AF})$  independent of  $AC \sim \text{Normal}(\mu_{AC}, \sigma^2_{AC})$ , then  $T \sim \text{Normal}(\mu_T, \sigma^2_T)$  where  $\mu_T = \mu_{AF} + \mu_{AC}$  and  $\sigma^2_T = \sigma^2_{AF} + \sigma^2_{AC}$ . Similarly for the gamma distribution,  $T \sim \text{Gamma}(\alpha_T, \lambda_T)$  and  $AC \sim \text{Gamma}(\alpha_{AC}, \lambda_{AC})$ , with  $\alpha_{AC} < \alpha_T$ , then  $AF \sim \text{Gamma}(\alpha_T - \alpha_{AC}, \lambda_T)$ .

As discussed in Chapter II, anti-corrosive thickness measurements must be taken after the AC layers are applied. If the AC measurements are not taken before the AF layers are applied, then they cannot be taken later. Furthermore, there exists limited sample data from measurements taken after just the AC layers have been applied. The reason AC measurements are difficult to collect is because the AC layers are still soft, and not completely dry, when the AF layers are applied. Shims must be used when measuring AC layer thickness. TT thickness measurements, however, may be taken at any time and relatively quickly, thus they are considered inexpensive and may be plentiful, if desired. Thus, for realistic simulations the sample sizes of the AC data sets vary between 50 and 3000, whereas the sizes of the TT data sets vary between 500 and 6000. To ensure each set of generated data is adequately represented, 320 sets of data are randomly generated for each selected sample data size.

## B. MEASURES OF EFFECTIVENESS

The data simulations are conducted in multiple Microsoft Excel spreadsheets, then the simulated data is analyzed using S-Plus. Ultimately, the resultant estimate of the AF thickness cdf must be recorded for each run of a simulation. The distribution of AF is estimated by partitioning [0,70 mils] into equally spaced values incremented by one mil.

For each simulation, there are 320 sets of AF cdf estimates, one for each set of randomly generated data.

The speed of convergence of the EM algorithm is an issue in the effectiveness of the estimator. Initially, 500 iterations of the EM algorithm are used on each set of generated data to determine the speed of convergence. There are two items that will be of use in determining the algorithm convergence speed. First, the value of log likelihood objective function will be collected at the end each of the EM algorithm iterations. The EM algorithm will converge to a local maximum, or minimum of the negative log likelihood, of the objective function. A second item to evaluate convergence of the estimator is “How much is each iteration of the EM algorithm changing the estimated AF distribution?” One way to determine this is to collect the sum of the absolute values of the differences, for the 71 estimated values of the AF distribution, at each subsequent iteration. This value will be critical in determining algorithm convergence.

In evaluating its effectiveness of the estimator for different sample sizes, the mean square error (MSE) is estimated by taking the average squared deviances or the differences of the estimated AF cdf and the actual AF cdf used to generate the data, evaluated at each thickness (i.e. 1, 2, 3, ..., 70). The components of mean square error are the bias and the variance of the estimator. Let  $\hat{p}$  be an estimate of  $p$ . Then, the mean square error of  $\hat{p}$  is decomposed into the variance and bias terms as follows:

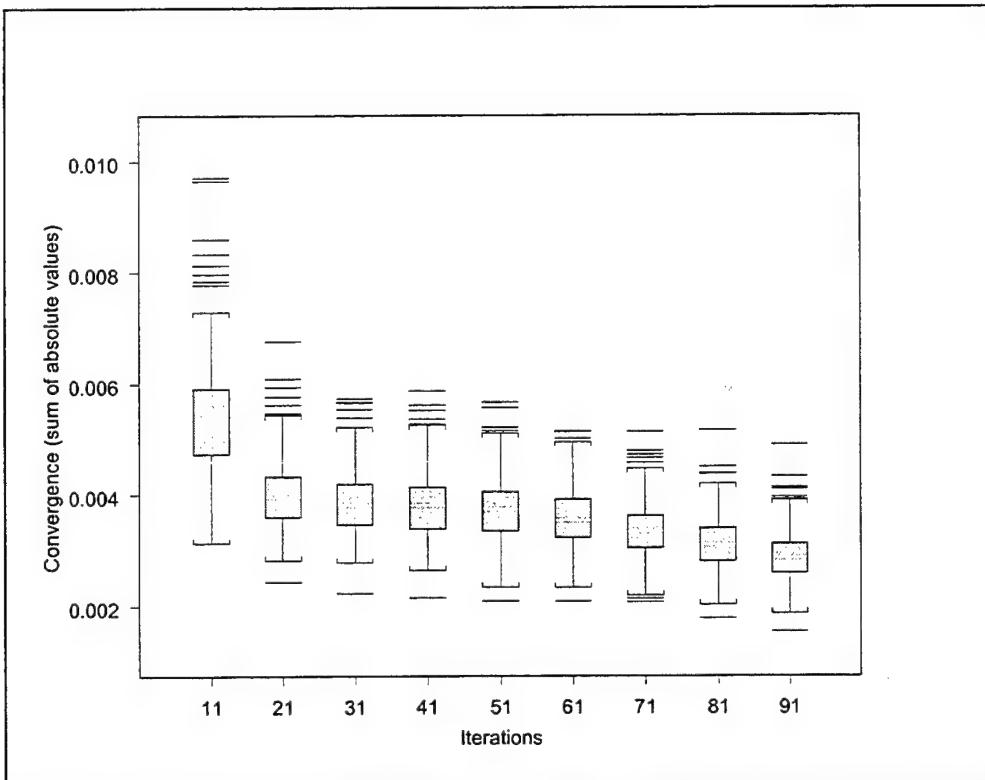
$$\begin{aligned}
MSE &= E[(\hat{p} - p)^2] \\
&= E[((\hat{p} - E(\hat{p})) + (E(\hat{p}) - p))^2] \\
&= E[(\hat{p} - E(\hat{p}))^2] + (E(\hat{p}) - p)^2 \\
&= \text{Variance}(\hat{p}) + (\text{Bias}(\hat{p}))^2.
\end{aligned}$$

Both bias and variance must be considered. The bias is estimated by the average deviance and the variance is estimated by the usual sample variance of deviances. We note that the standard deviation of deviance of the estimator from the true distribution is equal to the standard deviation of the estimator.

## C. RESULTS

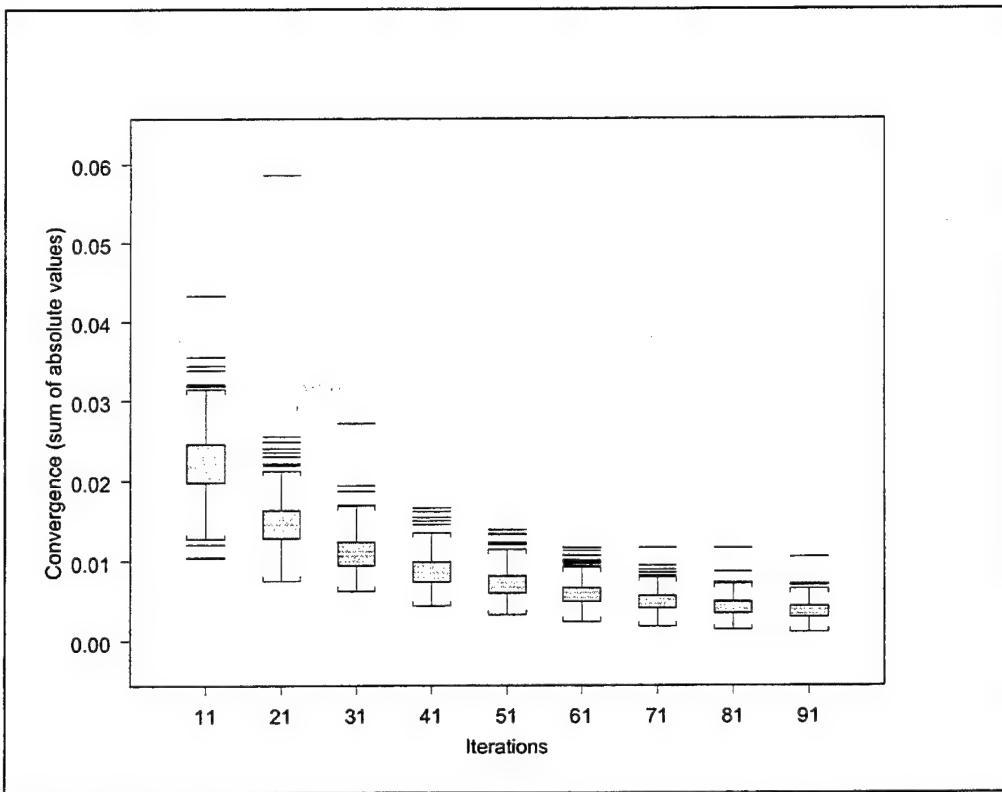
### 1. Speed of EM Algorithm Convergence

To fully analyze the effectiveness of the estimator, its properties must be individually evaluated. One property of the estimator is speed of algorithm convergence. As discussed, the estimator is based upon the notoriously slow EM algorithm. Although there are many methods to determine the speed of algorithm convergence, the most obvious appears to be to study the differences in the estimate of the anti-fouling distribution at each subsequent EM algorithm iteration. In Figures 9 and 10, the sum of the absolute values of the differences, for the entire 71 estimated values of the AF distribution, is shown for each subsequent iteration. Noting the scale on the y-axis of each figure, the algorithm converges very quickly in its estimate of the anti-fouling distribution. Also, the convergence of the algorithm improves when more observations are available.



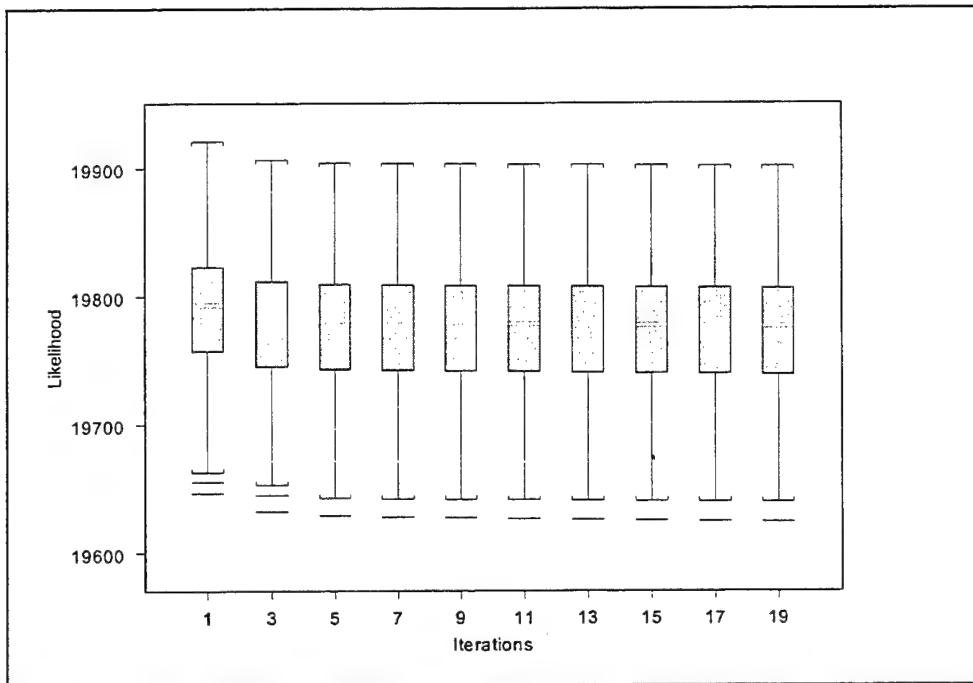
**Figure 9. Sum of Absolute Differences for Subsequent EM Algorithm Iterations. Gamma Random Variables With 100 AC Observations/6000 TT Observations.**

Figure 9 illustrates the rapid convergence of the estimator when 100 AC observations and 6000 TT observations are available. When fewer observations are available, the estimator still converges very quickly as shown in Figure 10.

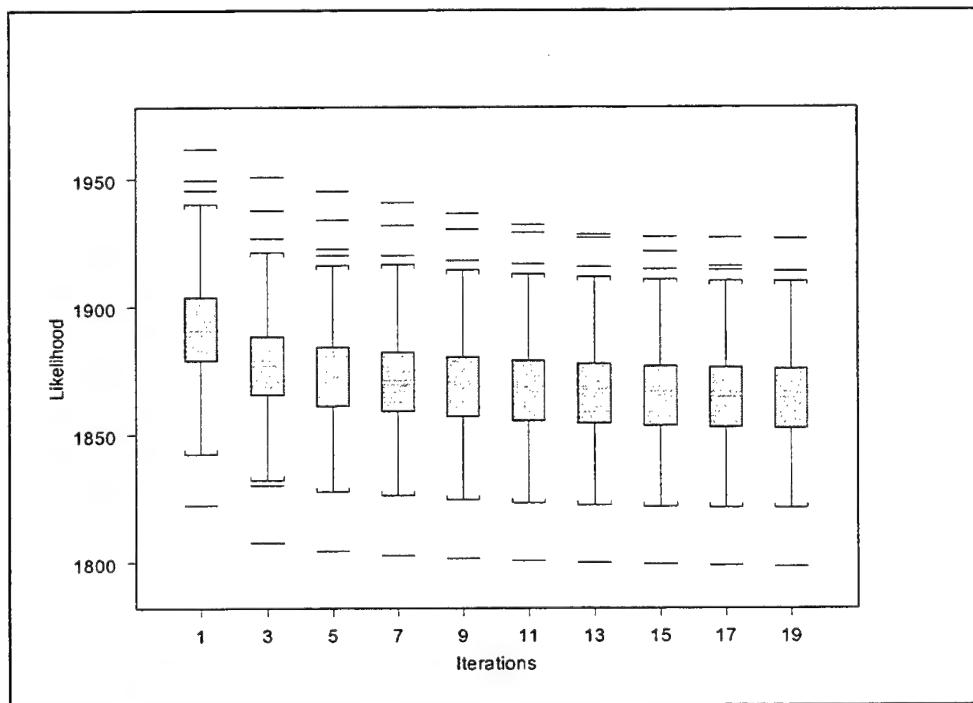


**Figure 10. Sum of Absolute Differences for Subsequent EM Algorithm Iterations.  
Gamma Random Variables With 100 AC Observations/500 TT Observations.**

Another measure of algorithm convergence relates to the behavior of the objective function. The objective function, expressed as the minimization of the negative of the log likelihood function, is viewed at subsequent iterations to analyze its speed of convergence. The algorithm converges on the local maximum likelihood estimator in a very small number of iterations, as shown in Figures 11 and 12.



**Figure 11. Minimization of the Log Likelihood Objective Function for EM Algorithm Iterations. Gamma Random Variables With 100 AC/6000 TT Observations.**

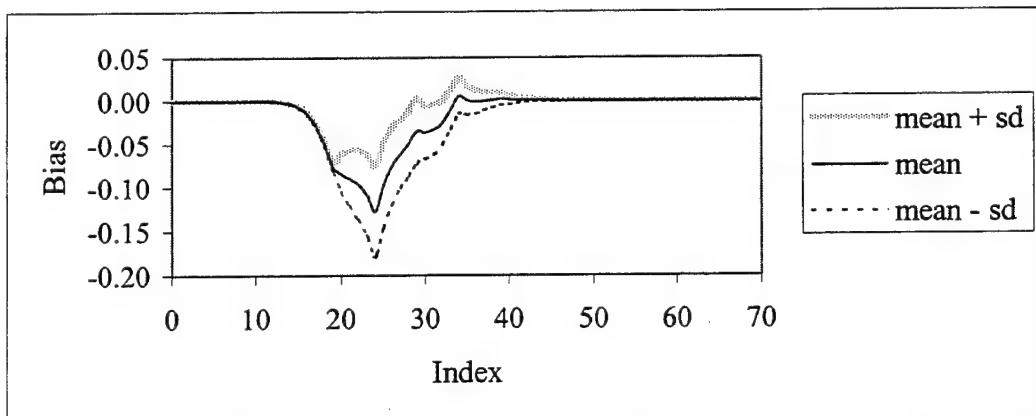


**Figure 12. Minimization of the Log Likelihood Objective Function for EM Algorithm Iterations. Gamma Random Variables With 100 AC/500 TT Observations.**

## 2. Bias and Variance of the Estimator

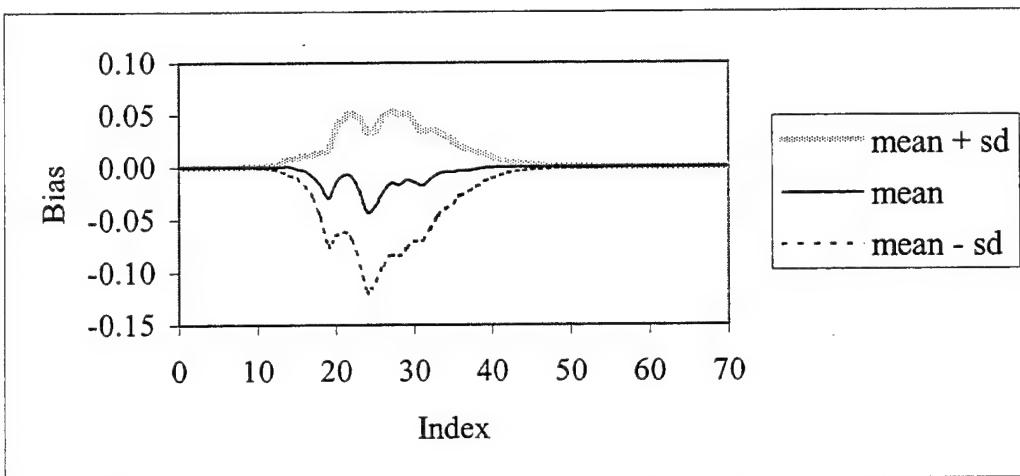
Both bias and variance must be considered when evaluating the estimator.

Figures 13 and 14, with data generated from Gamma distributions, show the estimated bias, with the estimated bias plus or minus one standard deviation of the deviance shown on the outer two lines. The index for these figures represents the [0, 70 mil] bins at which the anti-fouling distribution is computed. The center of the graphs, with bias equal to 0.0, represents an estimator that is always equal to the true anti-fouling distributions.



**Figure 13. EM Algorithm AF Estimate With Gamma Generated Random Observations (Sample Sizes: 100 AC and 6000 TT)**

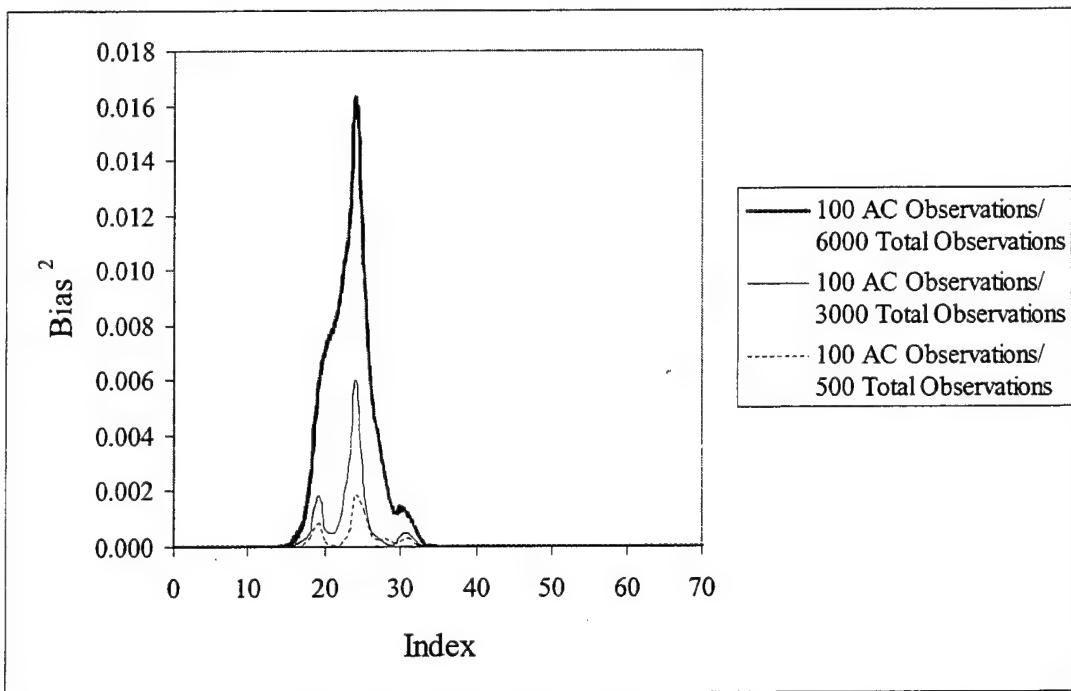
In Figure 13, with 100 AC observations and 6000 TT Gamma random observations, the estimator exhibits small variance and a relatively large negative bias. The negative bias may be attributed to the small number of AC observations available as compared to the large number of TT observations, whereas the small variance is simply due to the large number of TT observations available.



**Figure 14. EM Algorithm AF Estimate With Gamma Generated Random Observations (sample sizes: 100 AC and 500 TT)**

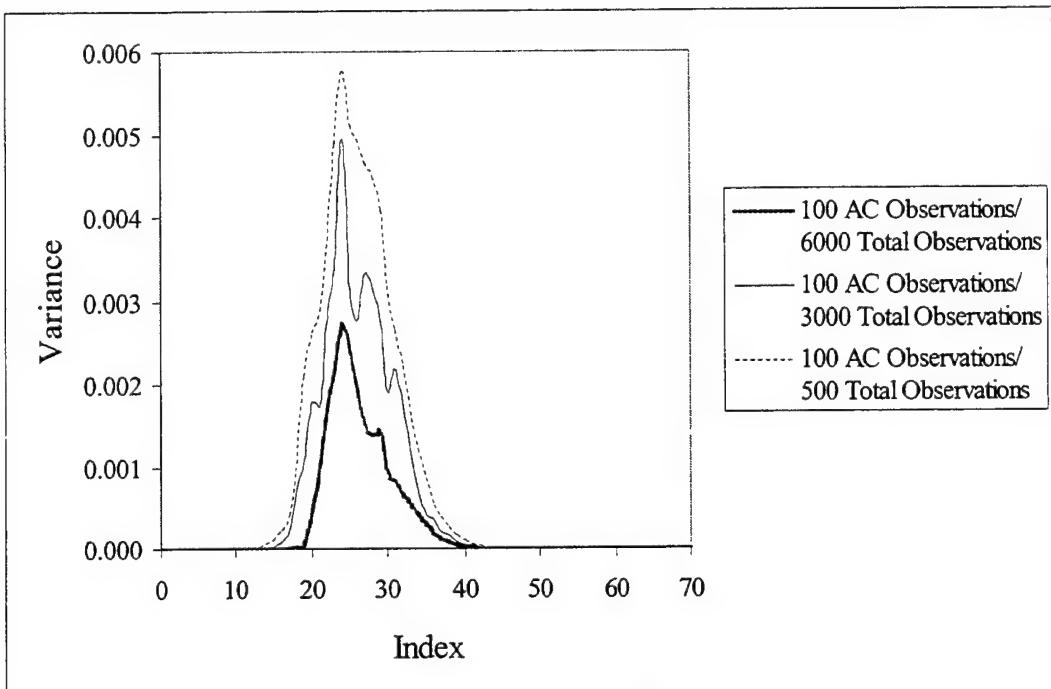
In contrast, Figure 14, with 100 AC observations and 500 TT Gamma random observations, the estimator exhibits large variance and a comparatively small negative bias. Here, the variance is attributed to the relatively small number of TT samples analyzed.

Clearly, the bias and variance are linked and vary in contribution to the estimator in accordance with the number of each type of observation available. The bias and the variance may also be viewed directly for different sample sizes, which will allow for the study of their contribution to the estimator and allow for appropriate analysis of the estimator's performance. Viewed directly, as the number of AC observations remains constant and the number of TT observations is increased, the bias of the estimator increases as shown in Figure 15, yet the variance decreases as shown in Figure 16.



**Figure 15. Bias of AF Estimator With 100 AC Gamma Random Observations and 500/3000/6000 TT Gamma Random Observations.**

In Figure 15, a comparison of the estimator bias is made when 100 AC observations are available. Note that the bias, as illustrated in Figure 15, is much less when AC and TT sample sizes are closer together. Regardless of underlying distributions or sample sizes studied, this is a property of the estimator.

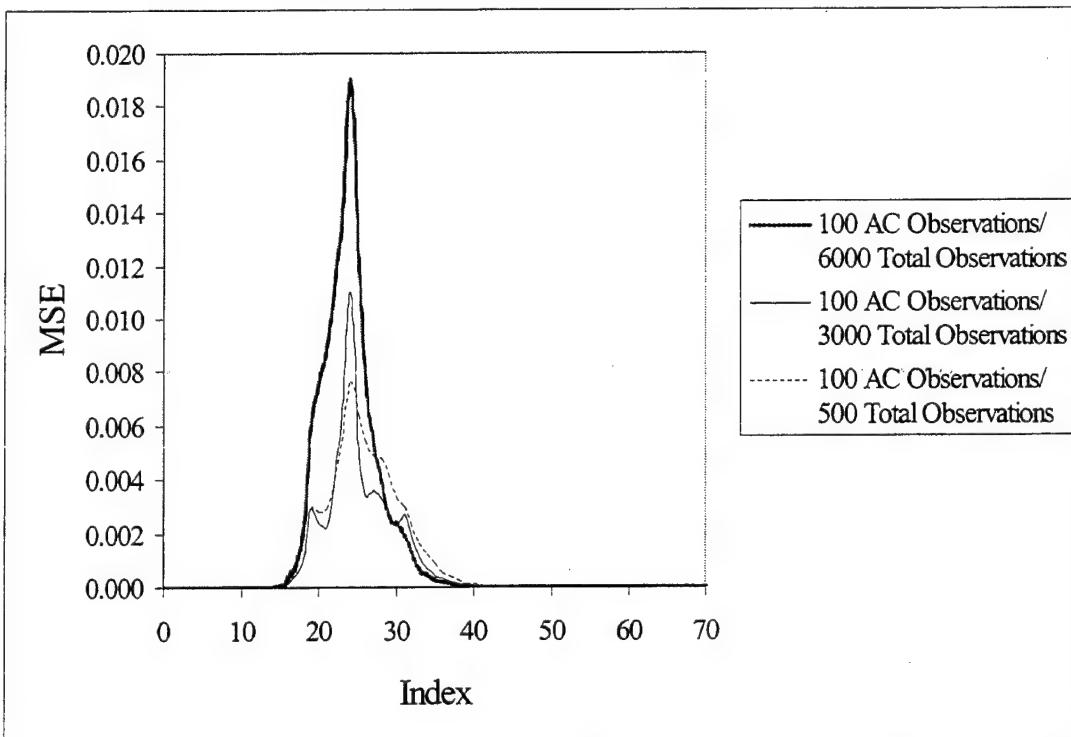


**Figure 16. Variance of AF Estimator With 100 AC Gamma Random Observations and 500/3000/6000 TT Gamma Random Observations.**

Note that the variance, as illustrated in Figure 16, is much less when more TT observations are available. Again, this is a property of the estimator that remains regardless of sample size or underlying distribution. Hence, the estimator is best evaluated by its Mean Square Error.

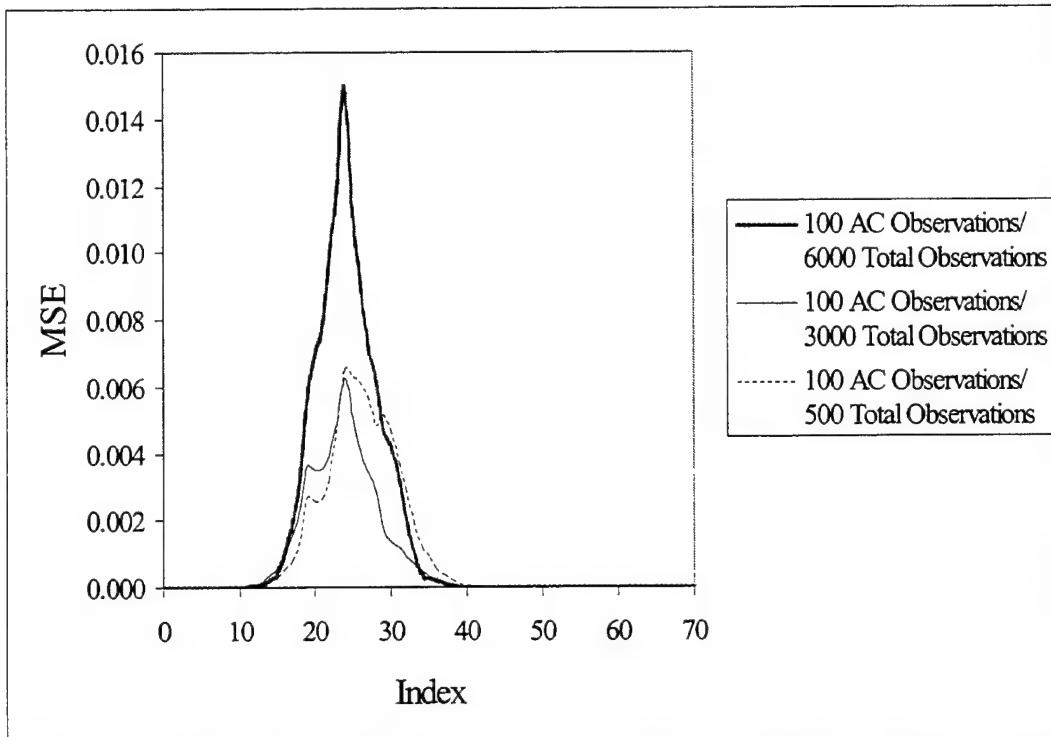
### 3. Mean Square Error of the Estimator

Due to the simultaneous changes in bias and variance, MSE appears to be the best means to evaluate the estimator. In Figure 17, the MSE shows that when 100 AC observations are available the estimator performs best with just 500 TT observations.



**Figure 17. MSE of AF Estimator With 100 AC Gamma Random Observations and 500/3000/6000 TT Gamma Random Observations.**

It is clear from Figure 17 that the estimator performs best, in cases where just 100 Gamma AC observations are available, with fewer Gamma TT observations. This is attributed to the large bias in the estimator, illustrated in Figure 15, when the number of TT observations is significantly greater in proportion to the number of AC observations. Regardless of the underlying distribution of the observations, when 100 AC observations are available, having just 500 TT observations is better than having more TT observations for estimator performance. Random samples generated from the Normal distribution, shown in Figure 18, further illustrate this property of the estimator when utilizing Mean Square Error to compare estimator performance over different sample sizes.



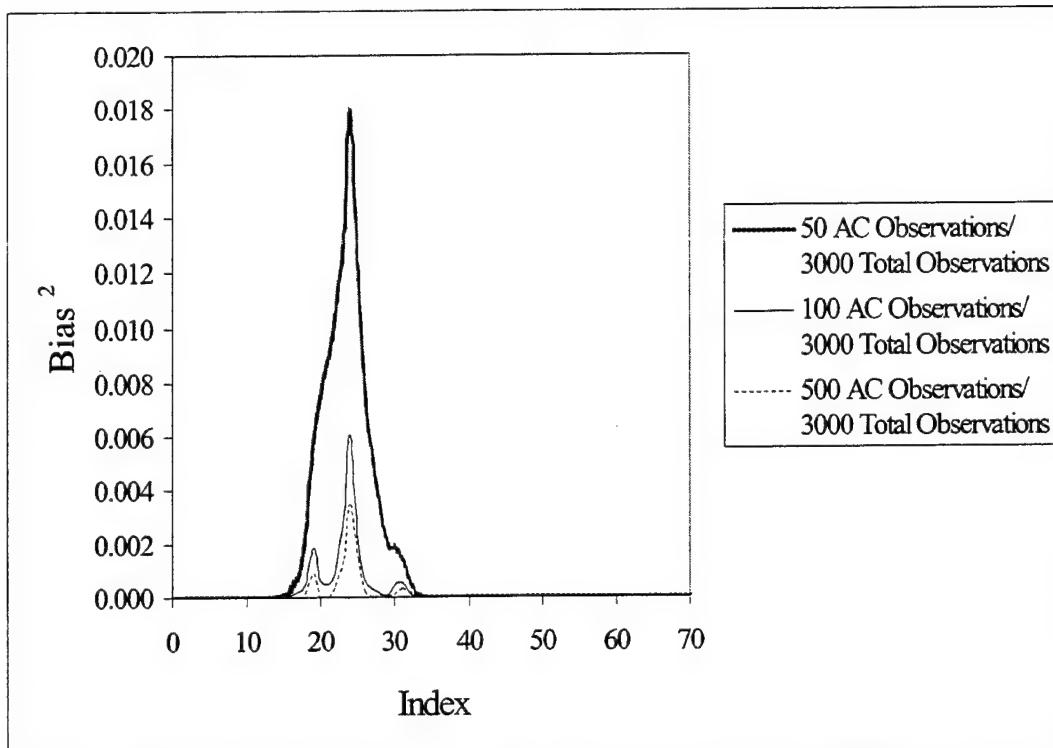
**Figure 18. MSE of AF Estimator With 100 AC Normal Random Observations and 500/3000/6000 TT Normal Random Observations.**

The underlying distribution for the observations in Figure 18 is Normal, however the results are similar to those from Gamma distribution observations. When just 100 AC observations are available, having a very large TT sample size does not help the accuracy of the anti-fouling estimator due to the large associated bias factor.

#### 4. Estimator Performance as a Function of Available AC Observations

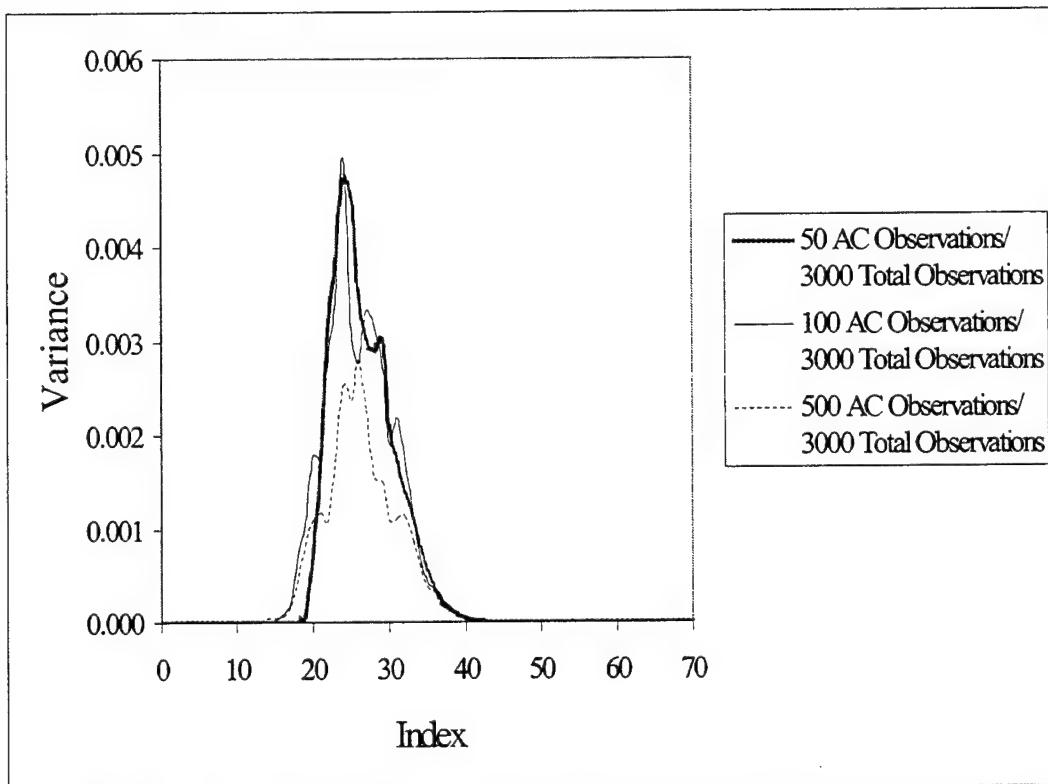
The bias error associated with having a small number of AC observations appears to hinder the estimator. The apparent solution is to increase the number of AC observation. As discussed, however, it is difficult and expensive to collect additional AC observations. Hence, the initial number of AC observations collected must be appropriate to reduce the bias of the estimator while still remaining cost effective. In order to study this property of the estimator, TT sample size is held constant and AC sample size is varied. Figure 19 illustrates that as the number of AC observations

increases, with TT sample size remaining constant, the associated bias of the estimator decreases.



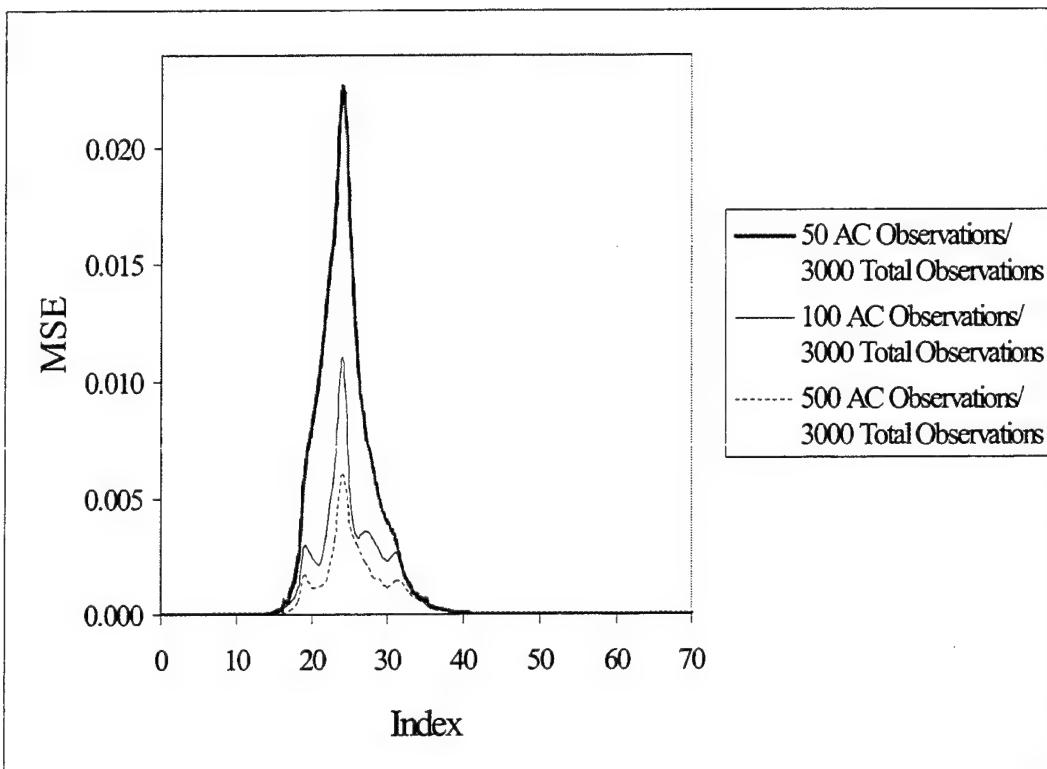
**Figure 19. Bias of AF Estimator With 50/100/500 AC Gamma Random Observations and 3000 TT Gamma Random Observations.**

Figure 19 shows that 500 AC observations, in conjunction with 3000 TT observations, will reduce the bias of the estimator and increase estimator performance. Regardless of the underlying distribution, the increasing of the number of AC observations will reduce estimator bias. Since bias is just one part of MSE, however, variance of different AC sample sizes is also studied. Figure 20 shows the effect of increasing the number of AC observations, while maintaining constant TT sample sizes, on the variance of the estimator.



**Figure 20. Variance of AF Estimator With 50/100/500 AC Gamma Random Observations and 3000 TT Gamma Random Observations.**

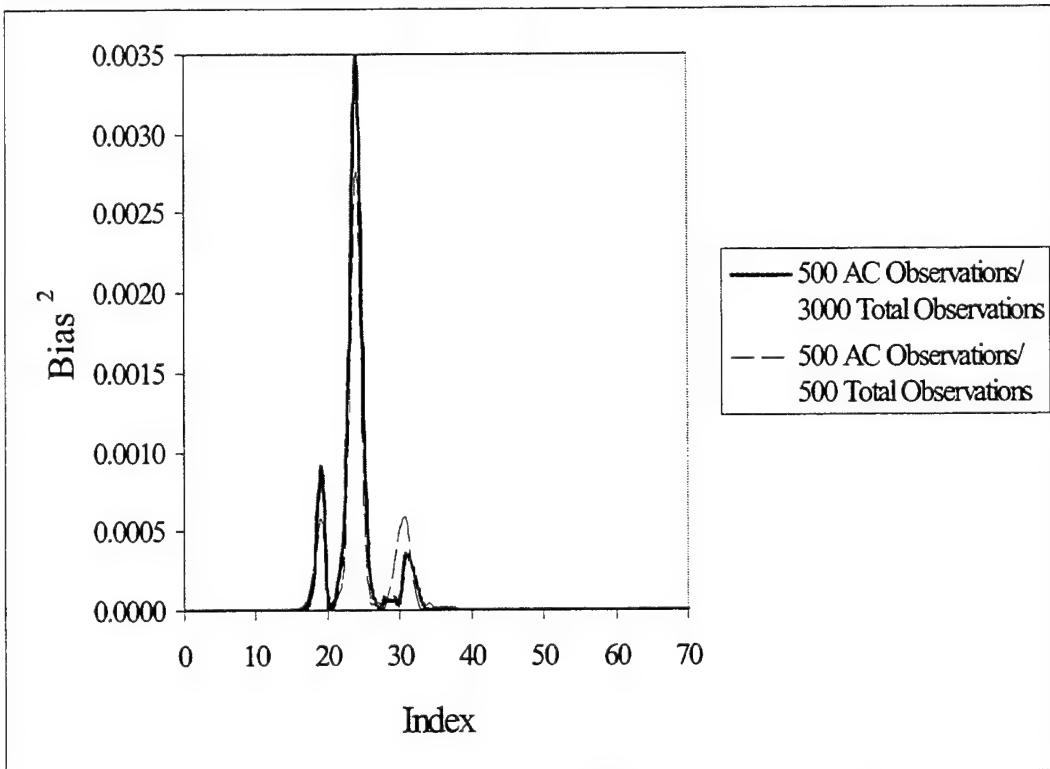
Increasing the number of AC observations, while maintaining a constant number of TT observations, decreases the variability of the estimator as shown in Figure 20. This result holds regardless of the underlying distribution of the observations. Thus, clearly the best performance of the estimator, in terms of MSE, is when more AC observations are available. Figure 21 shows the reduced Mean Square Error of the estimator with increasing sample sizes of AC observations.



**Figure 21. MSE of AF Estimator With 50/100/500 AC Gamma Random Observations and 3000 TT Gamma Random Observations.**

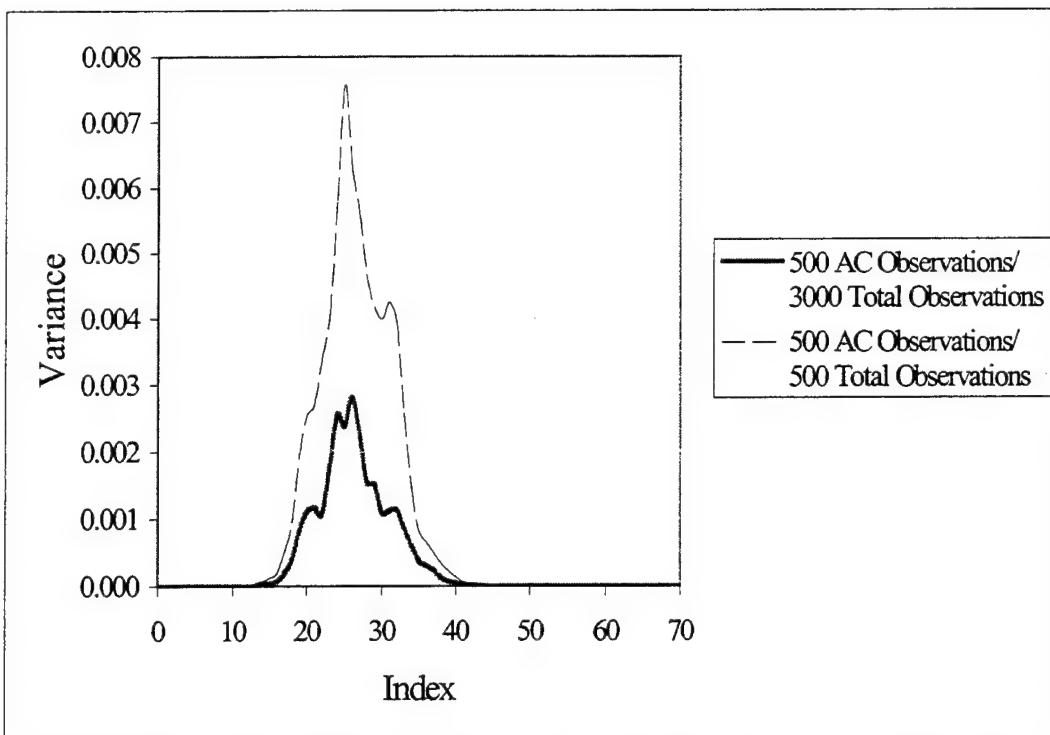
Figure 21, through Mean Square Error analysis, illustrates the benefit of increasing the number of AC observations available.

It is apparent that increasing the number of AC observations to 500 will decrease the bias of the estimator. Although the bias of the estimator is less when AC and TT sample sizes are closer together, if 500 or more AC observations are available the impact of the increased bias on the estimator is minimal. Illustrated in Figure 22, this is a property of the estimator regardless of underlying distributions analyzed.



**Figure 22. Bias of AF Estimator With 500 AC Gamma Random Observations and 500/3000 TT Gamma Random Observations.**

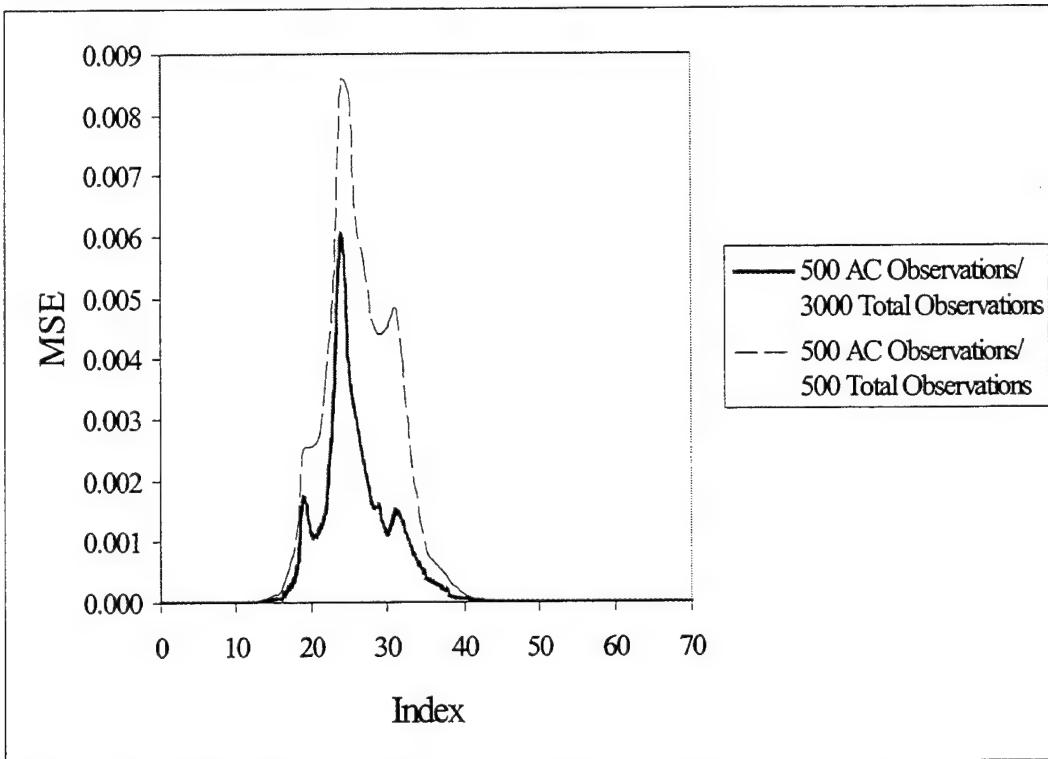
Figure 22 illustrates the decreasing impact of estimator bias when a minimum of 500 AC observations is available. Thus, increasing the number of TT observations when at least 500 AC observations are available will improve the estimator, since larger sample sizes are known to reduce estimator variance. The result of larger TT sample sizes being beneficial to the estimator is in contrast to previous results, which were generated with fewer AC observations. It is known that larger sample size, whether they are AC or TT observations, will reduce the variability of the estimator. Figure 23 shows the benefit of large sample sizes on the variance of the estimator.



**Figure 23. Variance of AF Estimator With 500 AC Gamma Random Observations and 500/3000 TT Gamma Random Observations.**

There is a significant reduction in variability of the estimator, as Figure 23 illustrates, when more observations are available.

As long as 500 AC observations are available, it is beneficial for the estimator to increase TT sample size. The major improvement in variability reduction shown in Figure 23, in conjunction with just a minor increase in bias shown in Figure 22, results in a much-improved Mean Square Error of the estimator. Figure 24 shows the benefit of an increased number of TT observations when 500 AC observations are available.



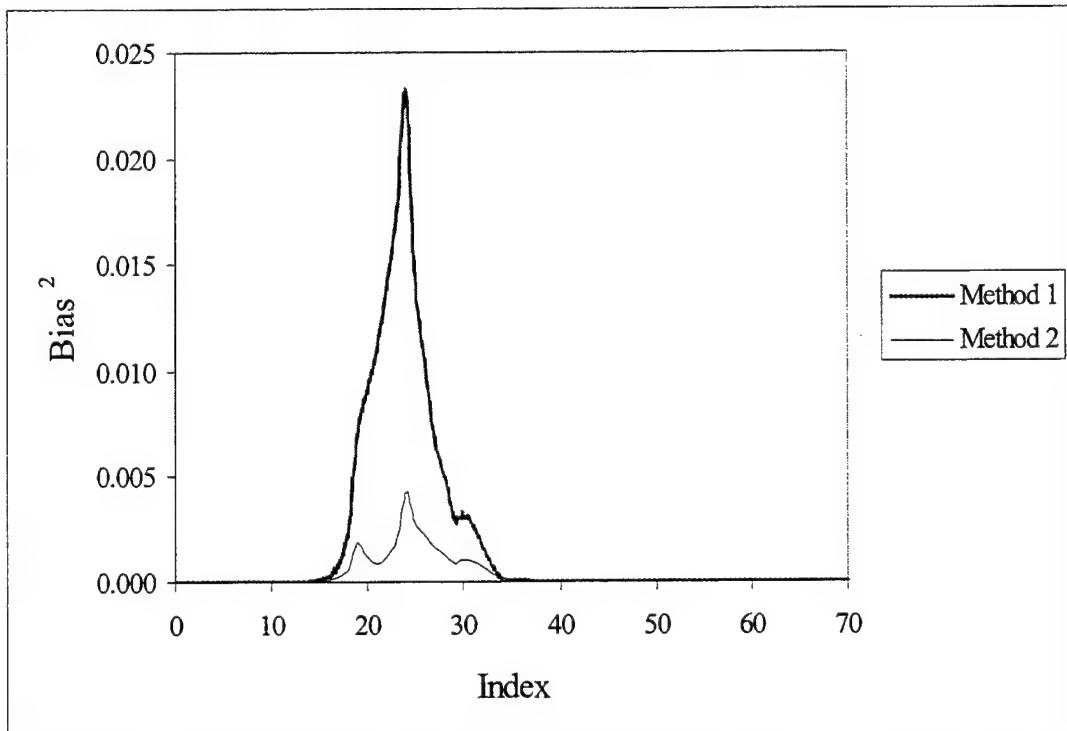
**Figure 24. MSE of AF estimator with 500 AC Gamma random observations and 500/3000 TT Gamma random observations.**

In cases where 500 AC observations are available, it is beneficial to the estimator to maximize the number of TT observations. Figure 24 shows the MSE of the estimator in one such case. Similar results are found with other underlying distributions.

### 5. An Alternative Estimator

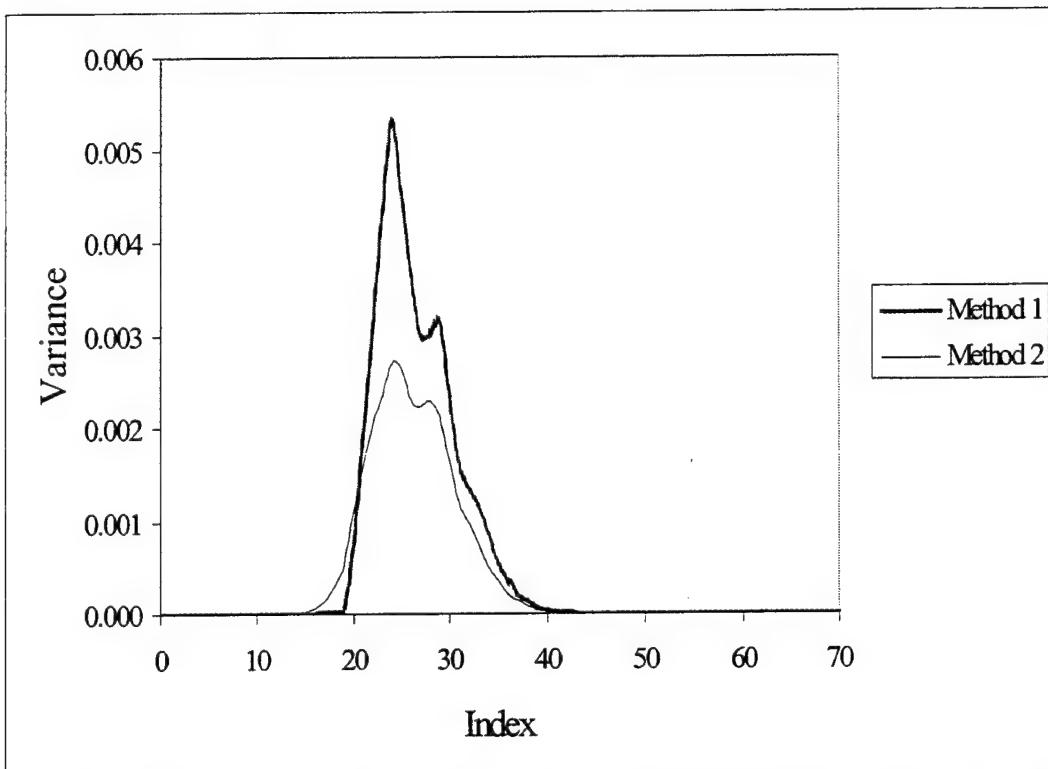
Many current data sets for AC observations, unfortunately, do not contain 500 AC observations. Since it is difficult and expensive to collect larger sample sizes of AC observations, it is desired to benefit from the estimator results illustrated in Figure 24 without the additional cost of increasing current actual data set sizes. An ad-hoc adjustment to the Whitaker AF estimator is proposed. It is an attempt to jointly capitalize on the small variance induced by large TT sample sizes on the estimator, yet still maintain the advantage in respect to the reduced bias error induced by small TT sizes

when the number of AC observations is low. Suppose the number of AC observations available is 50. The Whitaker estimator suggests a small TT sample size would be appropriate to reduce the large bias induced by larger TT sample sizes, thus reducing Mean Square Error. The ad-hoc adjustment to the estimator, however, is to break a large number of TT observations into smaller, equal data sets. Suppose 3000 TT observations are available, yet are divided into six sets of 500 TT observations each. The Whitaker estimator for the AF cdf is then applied six times when using the ad-hoc method, re-using the 50 AC observations on each of the six TT data sets. The result is a set of six estimates of the AF cdf, which are then averaged together to form an ad-hoc AF cdf estimate based on the Whitaker method. Figures 25, 26, and 27 show the Whitaker estimator to the entire AC and TT data sets as “Method 1”, and the ad-hoc estimator as “Method 2”. The ad-hoc estimator is the average of six iterations of the Whitaker estimator on smaller TT data sets, so reduced bias of this estimator is anticipated. Figure 25 illustrates the expected reduced bias of the ad-hoc estimator.



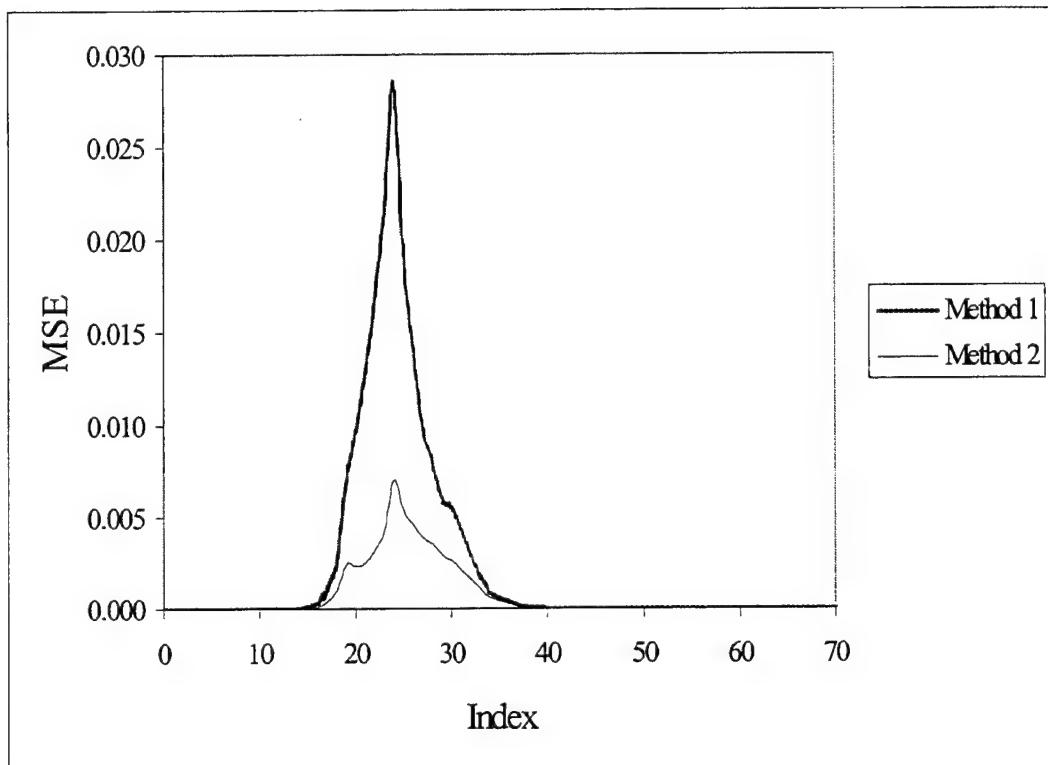
**Figure 25. Bias of Two AF Estimators With 50 AC Gamma Random Observations and 3000 TT Gamma Random Observations.**

The variance of the ad-hoc estimator, shown as “Method 2” in Figure 26, is also lower than the variance of the Whitaker estimator shown as “Method 1”. The reduced variance of the ad-hoc estimator is a powerful advantage for this method.



**Figure 26. Variance of Two AF Estimators With 50 AC Gamma Random Observations and 3000 TT Gamma Random Observations.**

The Mean Square Error plot of the Whitaker estimator and the ad-hoc estimator is shown in Figure 27. With its decreased bias and variability, the ad-hoc estimator provides an attractive method to apply the Whitaker estimator to currently available AC data sets, which are often relatively small in size.



**Figure 27. MSE of Two AF Estimators With 50 AC Gamma Random Observations and 3000 TT Gamma Random Observations.**

Although the ad-hoc estimator appears to be a viable alternative method of applying the Whitaker method, studying the properties of this ad-hoc estimator are beyond the scope of this thesis.

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## **V. DISCUSSION, CONCLUSIONS AND RECOMMENDATIONS**

As the Navy moves into the twenty-first century many financial challenges lie ahead. Reduction in the number of ships has become inevitable, as the Navy is required to maintain its surface fleet on a limited budget. There is a need to maintain operational commitments as well as reduce costs associated with drydocking ships. A major contributor to the total cost of drydocking is attributed to the maintenance and preservation of hull coating systems. By accurately estimating the AF layers thickness at any time, it is possible to determine whether sufficient AF paint remains on the hull. If so, then the coating system is salvageable and will only require simple repair and repainting at the next drydocking. If, however, too much AF is worn away during the operational cycle, then not only is the ship at risk for hull fouling, but also at drydocking the entire coating system needs to be removed at great expense and the ship repainted.

Wimmer (1997) develops a model to predict coating system wear. Ellis (1999) provides a model that determines the number of coats of AF paint to apply to an existing coating system, at a future interim drydock, to ensure the hull remains protected for a second operational cycle. Hinson (1999) suggests modifying the Wimmer and Ellis models to allow for the use of roughness data being collected during the operational cycle.

Previous work has provided significant improvements in extending the life of hull coating systems. There are, however, several limitations to the application of current hull coating distribution models. Research based on data collected by PERA(CV) focuses on the total coating thickness distribution. In the past, total paint thickness measurements have been used as a proxy for the distribution of the thickness of the AF paint layers

when determining if the existing coating system meets the hull coating requirements.

This thesis focuses on a new procedure for estimating the distribution of the AF layer thickness directly. The Whitaker method provides a means to more accurately assess the hull coating system of a ship.

#### **A. RECOMMENDATIONS FOR FUTURE DATA COLLECTION**

There exists limited sample data from measurements taken after just the AC layers have been applied, since collection of this data previously was not necessary. Anti-corrosive thickness measurements must be taken after the AC layers are applied and before the AF layers are applied, thus if the data sets do not exist it would be impossible to collect this data. Hence, when there is an opportunity to collect anti-corrosive data cheaply, when replacing the coating system at drydock, it should be collected. The Whitaker method clearly shows the advantage of having relatively large data sets. More accurate predictions of the current state of the AF layer thickness may be made if collection of anti-corrosive data becomes standard practice.

#### **B. ADVANTAGES OF THE WHITAKER (2000) MODEL**

Although the limited scope of this thesis allows it to serve only as a pilot study for more detailed analysis as more data becomes available, a potentially useful model for hull coating evaluation has been developed. The Whitaker model proves a non-parametric maximum likelihood estimator for the cumulative distribution function (cdf) of the AF layers. Previously, total paint thickness measurements have been used as a proxy for the distribution of the thickness of the AF paint layers when determining if the existing coating system meets the hull coating requirements. The Whitaker model provides an alternative that may be beneficial to the entire surface fleet.

The ideal method for determining the thickness of the anti-fouling layers is to develop a physical measuring tool which may be placed on the outer surface of a ship and utilized to determine the thickness of the anti-fouling paint at each point directly. Based on the results of many simulations, the Whitaker model would be an effective analytical tool in evaluating the effectiveness of any physical-measuring device developed in the future.

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## APPENDIX A. 1985 CVN-69 ANTI-CORROSIVE AND TOTAL DATA

<b>TT</b> <b>Thickness</b> <b>(mils)</b>	<b>Number of Observations</b>	<b>TT</b> <b>Thickness</b> <b>(mils)</b>	<b>Number of Observations</b>	<b>AC</b> <b>Thickness</b> <b>(mils)</b>	<b>Number of Observations</b>
0	0	36	54	0	0
1	0	37	67	1	0
2	0	38	70	2	0
3	0	39	69	3	0
4	0	40	81	4	0
5	0	41	96	5	0
6	0	42	68	6	0
7	0	43	60	7	3
8	0	44	75	8	7
9	0	45	51	9	9
10	0	46	56	10	4
11	0	47	30	11	7
12	0	48	31	12	22
13	0	49	8	13	15
14	0	50	17	14	13
15	0	51	9	15	7
16	0	52	10	16	14
17	0	53	7	17	13
18	0	54	4	18	5
19	1	55	1	19	5
20	2	56	3	20	1
21	2	57	5	21	1
22	1	58	1	22	1
23	3	59	2	23	0
24	5	60	0	24	0
25	3	61	2	over 25	0
26	6	62	1		
27	1	63	3		
28	6	64	0		
29	5	65	0		
30	14	66	1		
31	27	67	0		
32	31	68	1		
33	45	69	0		
34	51	70	1		
35	40	over 70	0		

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